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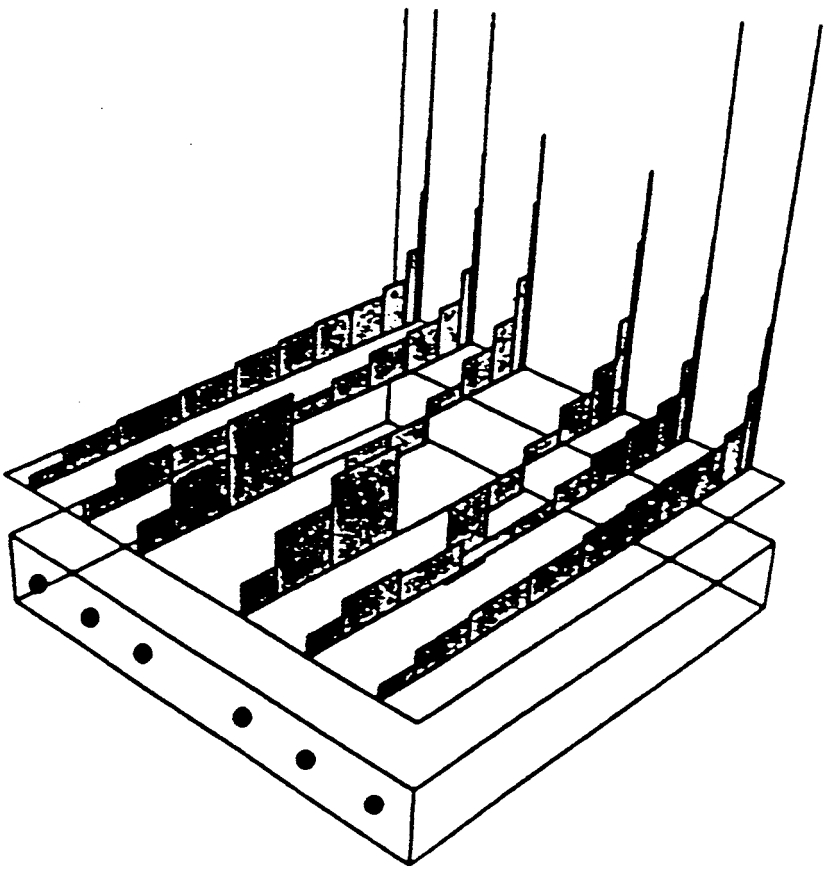
Florentin Smarandache

ONLY PROBLEMS, NOT SOLUTIONS!



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"ONLY PROBLEMS, NOT SOLUTIONS!", by Florentin Smarandache

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MATHEMATICAL PHILOSOPHY ?

instead of preface ...

The development of mathematics continues in a rapid rhythm, some unsolved problems are elucidated and simultaneously new open problems to be solved appear.

1. "Man is the measure of all things". Considering that mankind will last to infinite, is there a terminus point where this competition of development will end? And, if not, how far can science develop: even so to the infinite? That is . . .

The answer, of course, can be negative, not being an end of development, but a period of stagnation or of small regression. And, if this end of development existed, would it be a (self) destruction? Do we wear the terms of self-destruction in ourselves? (Does everything have an end, even the infinite? Of course, extremes meet.)

I, with my intuitive mind, cannot imagine what this infinite space means (without a beginning, without an end), and its infinity I can explain to myself only by means of a special property of space, a kind of a curved line which obliges me to always come across the same point, something like Moebus Band, or Klein Bottle, which can be put up/down (!)

I am not a specialist in physics, astronomy or philosophy, and I institute the infinite only theoretically

--from the point of view of mathematics (of course unilaterally).

2. Mathematics knows a high degree of abstraction, idealization, and generalization. And I ask, is it possible to build a pure mathematical model for society? You will answer, "it would be too rigid". And you are right because the non-elastic systems stop the progress. But I should replay, "they would be based upon logic". In the future could we become human robots, having the time programmed to the second (or even thousandth of a second!), elements of a mathematical-cybernetical system?

3. How do you imagine mathematics over 1,000 years? What about 1,000,000 year? (I exaggerate, of course.) What other new branches will appear? (Some will be ended, out of date?) (I'm not a futurist [Toepler]).

You may consider these questions too general, or too easy, but what can be more difficult than trying to say a maximum of ideas with a minimum of words? You are right, sometimes a too general idea becomes a common one. Maybe you think that asking questions is easy, but let me contradict you. (R. K. Guy said that asking questions is an art.) And after all, aren't the theories born, at their turn, from questions? (Maybe in this essay the questions are too direct, you are right again.)

4. If we consider "Math (t)", the development of mathematics at a time "t" (considered from the appearance of life on Earth) then

$$(\exists)\ L = \lim_{t \to \infty} \text{Math}\ (t)\ ?$$

And if it is yes, what is it equal with? With ∞ ? In case of total self-destruction should we obtain L = 0? And if life would reappear and the development would start again then should we draw the conclusion that $(\nexists)$ L ? (cyclical development).

5. In case of a total (self) destruction and reappearance of life on Earth, how would they call the Pythagoras' theorem, Euclid's geometry, Birkoff's axioms, Erdös' open problems, and so on? Will mankind pass through the same phases of development? Or, if it would exist, another civilization at the same time, how should they call these results? And how should we call them if these two (or more) civilizations united? (I have arrived in the field of history and of mathematical philosophy, which is not the purpose of this paper.) (All these questions can be also extended to other fields of knowledge.)

I can imagine computers with fantastic memories having the whole mathematics divided like a mosaic: this theorem belongs to X, the other one to Y, this sentence belongs to everybody, the other sentence belongs to nobody--the one who

will invent has not been born yet, but he will be born! A real dictionary of names and ideas, science divided in a finite (but, however, infinite) number of cells, each of them having a strict delimitation, with its own history, and the future generations will add new and new cells.

Although the applied mathematics, the integral calculus, the operator theory are the queens, the primitive arithmetic still fascinates the world because of its apparent elementary problems--very easy to be understood by everybody, but . . .

Why is mankind still interested in these easy problems, of mathematical juggler? I think that it is possible thanks to their simplicity in exposure. ("Ah, it's easy", one says, and to solve it you discover that you plunge into a labyrinth. And, hence, appears a paper: "On a conjecture . . .", "On the unsolved problem . . ." etc.)

I am sure that the "unsolved" problems presented in these pages will be (or have already been before the appearance of this essay) easy for many mathematicians, but for me they were an obsessions. W. Sierpiński was optimistic when he said that if mankind lasted then all these unsolved problems would be solved.

All the material in this paper is original in the author's opinion. He wanted to gather in this paper a variety of material, for the sake of an armony of contraries.

"I want to be a mathematician", as P. R. Halmos, and for the I began to play: rebus + mathematics, literature + mathematics, and even rebus + literature! So, please, do not wonder at this essay.

References

[1] R. K. Guy, Unsolved problems in number theory, New York, Heidelberg, Berlin: Springer-Verlag, 1981, p. vii.

[2] P. R. Halmos, I want to be a mathematician, An Automathography, Berlin, Heidelberg, New York, Tokyo: Lange & Springer, 1985.

[3] C. Truesdell, An idiot's fugitive essays on sciences, Methods, Criticism, Training, Circumstances: Lange & Springer, 1984.

UNSOLVED PROBLEM: 1

Find all integer sequences $\{a_n\}_{n\in N^*}$ defined as follows:

(I) $(\forall) i\in N^*$, $(\exists) j, k\in N^*$, $i\neq j\neq k\neq i$, such that

$a_i \equiv a_j \pmod{a_k}$.

(II) $(\forall) i\in N^*$, $(\exists) j, k\in N^*$, $i\neq j\neq k\neq i$, such that

$a_j \equiv a_k \pmod{a_i}$.

UNSOLVED PROBLEM: 2

Let $d > o$. Questions:

(a) Which is the maximum number of points included in a plane figure (generally: in a space body) such that the distance between any two points to be greater or equal than d?

(b) Which is the minimum number of points $\{A_1, A_2, \ldots\}$ included in a plane figure (generally: in a space body) such that if it includes another point A then there be an A_i with $|AA_i| < d$?

UNSOLVED PROBLEM: 3

(a) Let $a_1, \ldots, a_n$ be distinct digits of the set {0, 1, 2, ..., 9}, for a given n, $1 \leq n \leq 9$. How many distinct primes can we make up with all these digits? More generally: when $n \in N^*$ and $a_1, \ldots, a_n$ are distinct positive integers.

(b) Let $a \in \{0, 1, \ldots, 9\}$. How many digits of a does the n-th prime contain? But n! ? But n^n ? More generally: when $a \in N$.

Comment

"The sizes P_n, n!, n^n have jumps when $n \rightarrow n + 1$, hence the analytical expressions are approximate only. Moreover, the results depend on the exact (and not approximate) value of these sizes" (E. Grosswald [1]).

"(a) can be solved quickly on a modern computer" (R. K. Guy [2]).

References

[1] E. Grosswald, University of Pennsylvania, Philadelphia, USA. Letter to the Author; August 3, 1985.

[2] Richard K. Guy, University of Calgary, Alberta, Canada. Letter to the Author; November 15, 1985.

UNSOLVED PROBLEM: 4

Digital sequences:

(This a particular case of sequences of sequences.)

General definition:

in any numeration base B, for any given infinite integer or rational sequence S_1, S_2, S_3, ..., and any digit D from 0 to B-1

it's built up a new integer sequence witch

associates to S_1 the number of digits D of S_1 in base B,

to S_2 the number of digits D of S_2 in base B, and so on...

For exemple, considering the prime number sequence in base 10, then the number of digits 1 (for exemple) of each prime number following their order is: 0,0,0,0,2,1,1,1,0,0,1,0,...

(the digit-1 prime sequence).

Second exemple if we consider the factorial sequence n! in base 10, then the number of digits 0 of each factorial number following their order is: 0,0,0,0,0,1,1,2,2,1,3,...

(the digit-0 factorial sequence).

Third exemple if we consider the sequence n^n in base 10, n=1,2,. then the number of digits 5 of each term 1^1, 2^2, 3^3,..., following their order is: 0,0,0,1,1,1,1,0,0,0,...

(The digit-5 n^n sequence)

Construction sequences:

(This a particular case of sequences of sequences.)

General definition:

in any numeration base B, for any given infinite integer or rational sequence $S_1, S_2, S_3, \ldots$, and any digits $D_1, D_2, \ldots, D_k$ ($k < B$), it's built up a new integer sequence such that each of its terms $Q_1 < Q_2 < Q_3 < \ldots$ is formed by these digits $D_1, D_2, \ldots, D_k$ only (all these digits are used), and matches a term S_i of the previous sequence.

For exemple, considering in base 10 the prime number sequence, and the digits 1 and 7 (for exemple),
we construct a written-only-with-these-digits (all these digits are used) prime number new sequence: 17,71,...
(the digit-1-7-only prime sequence).

Second exemple, considering in base 10 the multiple of 3 sequence, and the digits 0 and 1,
we construct a written-only-with-these-digits (all these digits are used) multiple of 3 new sequence: 1011,1101, 1110,10011,10101,10110,11001,11010,11100,...
(the digit-0-1-only multiple of 3 sequence).

Study this sequence.

UNSOLVED PROBLEM: 6

Constructive set (of digits 1,2):

1,2,11,12,21,22,111,112,121,122,211,212,221,222,1111,1112,

1121,1122,1211,1212,1221,1222,21112112,2121,2122,2211,2212,

2221,2222,...

(Numbers formed by digits 1 and 2 only.)

Definition:

a1) 1, 2 belong to S;

a2) if a, b belong to S, then $\overline{ab}$ belongs to S too;

a3) only elements obtained by rules a1) and a2) applied a finite number of times belong to S.

Remark:

- there are 2^k numbers of k digits in the sequence, for k = 1, 2, 3, ... ;
- to obtain from the k-digits number group the (k+1)-digits number group, just put first the digit 1 and second the digit 2 in the front of all k-digits numbers.

UNSOLVED PROBLEM: 7

Constructive set (of digits 1,2,3):

1,2,3,11,12,13,21,22,23,31,32,33,111,112,113,121,122,123,

131,132,133,211,212,213,221,222,223,231,232,233,311,312,

313,321,322,323,331,332,333,...

(Numbers formed by digits 1, 2, and 3 only.)

Definition:

a1) 1, 2, 3 belong to S;

a2) if a, b belong to S, then $\overline{ab}$ belongs to S too;

a3) only elements obtained by rules a1) and a2) applied a finite number of times belong to S.

Remark:

- there are 3^k numbers of k digits in the sequence, for k = 1, 2, 3, ... ;
- to obtain from the k-digits number group the (k+1)-digits number group, just put first the digit 1, second the digit 2, and third the digit 3 in the front of all k-digits numbers.

UNSOLVED PROBLEM: 8

Generalizated constructive set:

(Numbers formed by digits d_1, d_2, ..., d_m only, all d_i being different each other, $1 <= m <= 9$.)

Definition:

a1) d_1, d_2, ..., d_m belong to S;

a2) if a, b belong to S, then $\overline{ab}$ belongs to S too;

a3) only elements obtained by rules a1) and a2) applied a finite number of times belong to S.

Remark:

- there are m^k numbers of k digits in the sequence, for k = 1, 2, 3, ... ;
- to obtain from the k-digits number group the (k+1)-digits number group, just put first the digit d_1, second the digit d_2, ..., and the m-th time digit d_m in the front of all k-digits numbers.

More general: all digits d_i can be replaced by numbers as large as we want (therefore of many digits each), and also m can be as large as we want.

Study these sequences.

UNSOLVED PROBLEM: 9

Rationalize the following fraction:

$$1 \Big/ \sum_{i=1}^{n} \sqrt[k_i]{a_i}.$$

UNSOLVED PROBLEM: 10

Mathematical Logic:

Is it true that for any question there is at least an answer? Reciprocally: Is any assertion the result of at least a question?

UNSOLVED PROBLEM: 11

Is it possible to construct a function which obtains all irrational numbers? But all transcendental numbers?

UNSOLVED PROBLEM: 12

Given n points in space, four by four noncoplanar, find a maximum m having the property that there are m points among n ones which constitute the vertexes of a convex polyhedron. [An extension of the following conjecture: Anyhow it chooses $2^{m-2} + 1$ points in plane, three by three noncolinear, there are among these m joints which are the vertexes of a convex polygon. (Ioan Tomescu, Problems of combinatorics and graph theory [Romanian], Bucharest, EDP, 1983.) For m = 5 the conjecture was proved; it was still proved that it can choose 2^{m-2} points in plane, three by three noncolinear, such that any m ones among these do not constitute the vertexes of a convex polygon.]

UNSOLVED PROBLEM: 13

What is the maximum number of circles of radius 1, at most tangential by twos, which are included into a circle of radius n? (Gamma 1/1986). This problem was generalized by Mihaly Bencze, who asks the maximum number of circles of radius $\varnothing(n)$, at the most tangential by twos, which are included into a circle of radium n, where φ is a function of n (Gamma 3/1986).

Study a similar problem for circles of radius 1 included into a given triangle (on Malfatti's problem). Similar questions for spheres, cones, cylinders, regular pyramids, etc. More generally: planar figures included into a given planar figure. And in the space, too.

UNSOLVED PROBLEM: 14

(a) Let $m \geq 5$ an integer. Find a minimum n (of course, n depends on m) having the property: anyhow it chooses n points in space, four by four noncoplanar, there exist m ones among these which belong to a surface of a sphere.

(b) Same question for an arbitrary spatial figure (for example: cone, cube, etc.).

(c) Similar problems in plane (for $m \geq 4$, and the points: three by three noncolinear).

UNSOLVED PROBLEM: 15

Consecutive sequence:

1,12,123,1234,12345,123456,1234567,12345678,123456789,

12345678910,1234567891011,123456789101112,

12345678910111213,...

How many primes are there among these numbers?

In a general form, the Consecutive Sequence is considered in an arbitrary numeration base B.

Reference:

Student Conference, University of Craiova, Department of Mathematics, April 1979, "Some problems in number theory" by Florentin Smarandache.

UNSOLVED PROBLEM: 16

Circular sequence:

1,12,21,123,231,312,1234,2341,3412,4123,12345,23451,34512,45123,51234,

1 2 3 4 5

123456,234561,345612,456123,561234,612345,1234567,2345671,3456712,...

6 7 ...

How many primes are there among these numbers?

UNSOLVED PROBLEM: 17

Symmetric sequence:

1,11,121,1221,12321,123321,1234321,12344321,123454321,

1234554321,12345654321,123456654321,1234567654321,

12345677654321,123456787654321,1234567887654321,

12345678987654321,123456789987654321,1234567891098765 4321,

123456789101098765 4321,12345678910111098765 4321,

1234567891011111098765 4321,...

How many primes are there among these numbers?

In a general form, the Symmetric Sequence is considered in an arbitrary numeration base B.

References:

Arizona State University, Hayden Library, "The Florentin Smarandache papers" special collection, Tempe, AZ 85287-1006, USA.

Student Conference, University of Craiova, Department of Mathematics, April 1979, "Some problems in number theory" by Florentin Smarandache.

UNSOLVED PROBLEM: 18

Deconstructive sequence:

1,23,456,7891,23456,789123,4567891,23456789,123456789,1234567891, ...

How many primes are there among these numbers?

References:

Arizona State University, Hayden Library, "The Florentin Smarandache papers" special collection, Tempe, AZ 85287-1006, USA.

UNSOLVED PROBLEM: 19

Mirror sequence:

1,212,32123,4321234,543212345,65432123456,7654321234567,

876543212345678,98765432123456789,109876543212345678910,

11109876543212345678910 11,...

Question: How many of them are primes?

UNSOLVED PROBLEM: 20

Permutation sequence:

12,1342,135642,13578642,13579108642,135791112108642,

135791113141210864 2,1357911131516141210864 2,

135791113151718161412108642,13579111315171920181614121086 42,...

Question: Is there any perfect power among these numbers?

(Their last digit should be:

either 2 for exponents of the form 4k+1,

either 8 for exponents of the form 4k+3, where k >= 0 .)

Conjecture: no!

Generalizated permutation sequence:

If g(n), as a function, gives the number of digits of a(n),

and F if a permutation of g(n) elements, then:

$a(n) = \overline{F(1)F(2)...F(g(n))}$.

UNSOLVED PROBLEM: 21

Digital sum:

0,1,2,3,4,5,6,7,8,9,1,2,3,4,5,6,7,8,9,10,2,3,4,5,6,7,8,9,10,11,

3,4,5,6,7,8,9,10,11,12,4,5,6,7,8,9,10,11,12,13,5,6,7,8,9,10,11,12,13,14

($d_s(n)$ is the sum of digits.)

Study this sequence.

UNSOLVED PROBLEM: 22

Digital products:

0,1,2,3,4,5,6,7,8,9,0,1,2,3,4,5,6,7,8,9,0,2,4,6,8,19,12,14,16,18,

0,3,6,9,12,15,18,21,24,27,0,4,8,12,16,20,24,28,32,36,0,5,10,15,20,25,...

($d_p(n)$ is the product of digits.)

Study this sequence.

UNSOLVED PROBLEM: 23

Simple numbers:

2,3,4,5,6,7,8,9,10,11,13,14,15,17,19,21,22,23,25,26,27,29,31,

33,34,35,37,38,39,41,43,45,46,47,49,51,53,55,57,58,61,62,65,

67,69,71,73,74,77,78,79,82,83,85,86,87,89,91,93,94,95,97,101,103,.

(A number n is called <simple number> if the product of its proper divisors is less than or equal to n.)

Generally speaking, n has the form:

n = p, or p^2, or p^3, or pq, where p and q are distinct primes.

Study this sequence.

References:

Student Conference, University of Craiova, Department of Mathematics, April 1979, "Some problems in number theory" by Florentin Smarandache.

UNSOLVED PROBLEM: 24

Pierced chain:

101,1010101,10101010101,101010101010101,1010101010101010101,

10101010101010101010101,1010101010101010101010101010101,...

$$(c(n) = 101 * 1\underbrace{0001}_{1}\underbrace{0001}_{2}\ldots\underbrace{0001}_{n-1}, \text{ for } n >= 1)$$

How many c(n)/101 are primes ?

References:

Arizona State University, Hayden Library, "The Florentin Smarandache papers" special collection, Tempe, AZ 85287-1006, USA.

Student Conference, University of Craiova, Department of Mathematics, April 1979, "Some problems in number theory" by Florentin Smarandache.

UNSOLVED PROBLEM: 25

Divisor products:

1,2,3,8,5,36,7,64,27,100,11,1728,13,196,225,1024,17,5832,19,

8000,441,484,23,331776,125,676,729,21952,29,810000,31,32768,

1089,1156,1225,10077696,37,1444,1521,2560000,41,...

($P_d(n)$ is the product of all positive divisors of n.)

Study this sequence.

UNSOLVED PROBLEM: 26

Proper divisor products:

1,1,1,2,1,6,1,8,3,10,1,144,1,14,15,64,1,324,1,400,21,22,1,

13824,5,26,27,784,1,27000,1,1024,33,34,35,279936,1,38,39,

64000,1,...

($p_d(n)$ is the product of all positive divisors of n but n.)

Study this sequence.

UNSOLVED PROBLEM: 27

Square complements:

1,2,3,1,5,6,7,2,1,10,11,3,14,15,1,17,2,19,5,21,22,23,6,1,26,

3,7,29,30,31,2,33,34,35,1,37,38,39,10,41,42,43,11,5,46,47,3,

1,2,51,13,53,6,55,14,57,58,59,15,61,62,7,1,65,66,67,17,69,70,71,2,

Definition:

for each integer n to find the smallest integer k such that nk is a perfect square..

(All these numbers are square free.)

Study this sequence.

UNSOLVED PROBLEM: 28

Cubic complements:

1,4,9,2,25,36,49,1,3,100,121,18,169,196,225,4,289,12,361,50,

441,484,529,9,5,676,1,841,900,961,2,1089,1156,1225,6,1369,

1444,1521,25,1681,1764,1849,242,75,2116,2209,36,7,20,...

Definition:

for each integer n to find the smallest integer k such that nk is a perfect cub.

(All these numbers are cube free.)

Study this sequence.

UNSOLVED PROBLEM: 29

m-power complements:

Definition:

for each integer n to find the smallest integer k such that nk is a perfect m-power (m => 2).

(All these numbers are m-power free.)

Study this sequence.

Reference:

Arizona State University, Hayden Library, "The Florentin Smarandache papers" special collection, Tempe, AZ 85287-1006, USA.

UNSOLVED PROBLEM: 30

Cube free sieve:

2,3,4,5,6,7,9,10,11,12,13,14,15,17,18,19,20,21,22,23,25,26,
28,29,30,31,33,34,35,36,37,38,39,41,42,43,44,45,46,47,49,50,
51,52,53,55,57,58,59,60,61,62,63,65,66,67,68,69,70,71,73,...

Definition: from the set of natural numbers (except 0 and 1):

- take off all multiples of 2^3 (i.e. 8, 16, 24, 32, 40, ...)
- take off all multiples of 3^3
- take off all multiples of 5^3

... and so on (take off all multiples of all cubic primes).

(One obtains all cube free numbers.)

Study this sequence.

UNSOLVED PROBLEM: 31

m-power free sieve:

Definition: from the set of natural numbers (except 0 and 1)

take off all multiples of 2^m, afterwards all multiples of 3^m, .

and so on (take off all multiples of all m-power primes, m >= 2).

(One obtains all m-power free numbers.)

Study this sequence.

UNSOLVED PROBLEM: 32

Irrational root sieve:

2,3,5,6,7,10,11,12,13,14,15,17,18,19,20,21,22,23,24,26,28,
29,30,31,33,34,35,37,38,39,40,41,42,43,44,45,46,47,48,50,
51,52,53,54,55,56,57,58,59,60,61,62,63,65,66,67,68,69,70,71,72,73,

Definition: from the set of natural numbers (except 0 and 1):

- take off all powers of 2^k, k >= 2, (i.e. 4, 8, 16, 32, 64, ...
- take off all powers of 3^k, k >= 2;
- take off all powers of 5^k, k >= 2;
- take off all powers of 6^k, k >= 2;
- take off all powers of 7^k, k >= 2;
- take off all powers of 10^k, k >= 2;

... and so on (take off all k-powers, k >= 2, of all square free numbers).

One gets all square free numbers by the following method (sieve):

from the set of natural numbers (except 0 and 1):

- take off all multiples of 2^2 (i.e. 4, 8, 12, 16, 20, ...)
- take off all multiples of 3^2
- take off all multiples of 5^2

... and so on (take off all multiples of all square primes);

one obtains, therefore:

2,3,5,6,7,10,11,13,14,15,17,19,21,22,23,26,29,30,31,33,34,
35,37,38,39,41,42,43,46,47,51,53,55,57,58,59,61,62,65,66,
67,69,70,71,... ,

which are used for irrational root sieve.

(One obtains all natural numbers those m-th roots, for any m >= 2, are irrational.)

Study this sequence.

References:

Arizona State University, Hayden Library, "The Florentin Smarandache papers" special collection, Tempe, AZ 85287-1006, USA.

Student Conference, University of Craiova, Department of Mathematics, April 1979, "Some problems in number theory" by Florentin Smarandache.

UNSOLVED PROBLEM: 33

Let a_1, a_2, ..., a_m be digits. Are there primes, on a base b, which contain the group of digits $\overline{a_1 \ldots a_m}$ into its writing? (For example, if $a_1 = 0$ and $a_2 = 9$ there are primes as 1<u>09</u>, 4<u>09</u>, 7<u>09</u>, 8<u>09</u>, ...) But n! ? But n^n ?

UNSOLVED PROBLEM: 34

Conjecture

Let $k \geq 2$ a positive integer. The diophantine equation:

$$y = 2\, x_1 x_2 \ldots x_k + 1$$

has an infinity of solutions of primes. (For example: $571 = 2 \cdot 3 \cdot 5 \cdot 19 + 1$, $691 = 2 \cdot 3 \cdot 5 \cdot 23 + 1$, or $647 = 2 \cdot 17 \cdot 19 + 1$, when $k = 4$, respectively, 3). (Gamma 2/1986).

UNSOLVED PROBLEM: 35

Let d_n be the distance between two consecutive primes, $d_n = \frac{1}{2}(p_{n+1} - p_n)$, $n = 1, 2, \ldots$ Does d_n contain an infinite number of primes?

Does d_n contain numbers of the form n! ? But of the form n^n ?

Let $i \in N^*$, and $d_n^{(i)} = \frac{1}{2}(p_{n+i} - p_n)$, and $d_{n,i}^{(j)} = \frac{1}{2}(d_{n+j}^{(i)} - d_n^{(i)})$ for $n = 1, 2, \ldots$

The same questions.

(Gamma 2/1986).

UNSOLVED PROBLEM: 36

Conjecture

Anyhow the points of a plane are colored with n colors, there exists a color which fulfills all distances [i.e., $(\exists)$ a color c, $(\forall)$ $d \geq o$, $(\exists)$ the points A, B colored in c, such that the line AB is colored in c and $|AB| = d$].

(The result would implicitly be true in space, too.)

UNSOLVED PROBLEM: 37

Let $k, n \in N^*$, $k < n$. We define a decreasing sequence of integers: $n_o = n$ and $n_{i+1} = \max \{p, p \mid n_i - k, p \text{ is a prime}\}$, for $i \geq 0$.

Find the length (the number of terms) of this sequence (Gamma 2-3/1987).

UNSOLVED PROBLEM: 38

Spreading to infinite van der Waerden's theorem: Is it possible to partition N^* into an infinity of arbitrary classes such that at least one class contain an arithmetic progression of ℓ terms ($\ell \geq 3$)?

Find a maximum ℓ having this property.

UNSOLVED PROBLEM: 39

(Inferior) prime part:

2,3,3,5,5,7,7,7,7,11,11,13,13,13,13,17,17,19,19,19,19,23,23,

23,23,23,23,29,29,31,31,31,31,31,31,37,37,37,37,41,41,43,43,

43,43,47,47,47,47,47,47,53,53,53,53,53,53,59, ...

(For any positive real number n one defines $p_p(n)$ as the largest prime number less than or equal to n.)

(Superior) prime part:

2,2,2,3,5,5,7,7,11,11,11,11,13,13,17,17,17,17,19,19,23,23,

23,23,29,29,29,29,29,29,31,31,37,37,37,37,37,37,41,41,41,

41,43,43,47,47,47,47,53,53,53,53,53,53,59,59,59,59,59,59,61,...

(For any positive real number n one defines $P_p(n)$ as the smallest prime number greater than or equal to n.)

Study these sequences.

Reference:

Arizona State University, Hayden Library, "The Florentin Smarandache papers" special collection, Tempe, AZ 85287-1006, USA.

UNSOLVED PROBLEM: 40

(Inferior) square part:

0,1,1,1,4,4,4,4,4,9,9,9,9,9,9,9,16,16,16,16,16,16,16,16,16,
25,25,25,25,25,25,25,25,25,25,25,36,36,36,36,36,36,36,36,36,
36,36,36,36,49,49,49,49,49,49,49,49,49,49,49,49,49,49,64,64,...

(The largest square less than or equal to n.)

(Superior) square part:

0,1,4,4,4,9,9,9,9,9,16,16,16,16,16,16,16,25,25,25,25,25,25,
25,25,25,36,36,36,36,36,36,36,36,36,36,49,49,49,49,49,49,
49,49,49,49,49,49,49,64,64,64,64,64,64,64,64,64,64,64,64,
64,64,81,81,...

(The smallest square greater than or equal to n.)

Study these sequences.

UNSOLVED PROBLEM: 41

(Inferior) cube part:

0,1,1,1,1,1,1,1,8,8,8,8,8,8,8,8,8,8,8,8,8,8,8,8,8,27,27,
27,27,27,27,27,27,27,27,27,27,27,27,27,27,27,27,27,27,27,
27,27,27,27,27,27,27,27,27,27,27,27,27,27,27,64,64,64,...

(The largest cube less than or equal to n.)

(Superior) cube part:

0,1,8,8,8,8,8,8,8,27,27,27,27,27,27,27,27,27,27,27,27,27,
27,27,27,27,27,27,64,64,64,64,64,64,64,64,64,64,64,64,64,
64,64,64,64,64,64,64,64,64,64,64,64,64,64,64,64,64,64,64,
64,64,64,64,64,125,125,125,..

(The smalest cube greater than or equal to n.)

Study these sequences.

UNSOLVED PROBLEM: 42

(Inferior) factorial part:

1,2,2,2,2,6,6,6,6,6,6,6,6,6,6,6,6,6,6,6,6,24,24,24,24,

24,24,24,24,24,24,24,24,24,24,24,24,24,...

($F_p(n)$ is the largest factorial less than or equal to n.)

(Superior) factorial part:

1,2,6,6,6,6,24,24,24,24,24,24,24,24,24,24,24,24,24,24,24,

24,24,120,120,120,120,120,120,120,120,120,120,...

($f_p(n)$ is the smallest factorial greater than or equal to n.)

Study these sequences.

UNSOLVED PROBLEM: 43

Double factorial complements:

1,1,1,2,3,8,15,1,105,192,945,4,10395,46080,1,3,2027025,2560,
34459425,192,5,3715891200,13749310575,2,81081,1961990553600,
35,23040,213458046676875,128,6190283353629375,12,...

(For each n to find the smallest k such that nk is a double factorial, i.e. nk = either 1*3*5*7*9*...*n if n is odd,
either 2*4*6*8*...*n if n is even.)

Study this sequence in interrelation with Smarandache function { S(n) is the smallest integer such that S(n)! is divisible by n).

UNSOLVED PROBLEM: 44

Prime additive complements:

1,0,0,1,0,1,0,3,2,1,0,1,0,3,2,1,0,1,0,3,2,1,0,5,4,3,2,1,0,

1,0,5,4,3,2,1,0,3,2,1,0,1,0,3,2,1,0,5,4,3,2,1,0,...

(For each n to find the smallest k such that n+k is prime.)

Remark: is it possible to get as large as we want but finite decreasing k, k-1, k-2, ..., 2, 1, 0 (odd k) sequence included in the previous sequence -- i.e. for any even integer are there two primes those difference is equal to it? I conjecture the answer is negative.

Reference:

Arizona State University, Hayden Library, "The Florentin Smarandache papers" special collection, Tempe, AZ 85287-1006, USA.

UNSOLVED PROBLEM: 45

Factorial quotients:

1,1,2,6,24,1,720,3,80,12,3628800,2,479001600,360,8,45,

20922789888000,40,6402373705728000,6,240,1814400,

1124000727777607680000,1,145152,239500800,13440,180,

30488834461171386050150400000,...

(For each n to find the smallest k such that nk is a factorial number.)

Study this sequence in interrelation with Smarandache function.

Reference:

Arizona State University, Hayden Library, "The Florentin Smarandache papers" special collection, Tempe, AZ 85287-1006, USA.

UNSOLVED PROBLEM: 46

Double factorial numbers:

1,2,3,4,5,6,7,4,9,10,11,6,13,14,5,6,17,12,19,10,7,22,23,6,

15,26,9,14,29,10,31,8,11,34,7,12,37,38,13,10,41,14,43,22,9,

46,47,6,21,10,...

($d_f(n)$ is the smallest integer such that $d_f(n)!!$ is a multiple of n.)

Study this sequence in interrelation with Smarandache function.

Reference:

Arizona State University, Hayden Library, "The Florentin Smarandache papers" special collection, Tempe, AZ 85287-1006, USA.

UNSOLVED PROBLEM: 47

Primitive numbers (of power 2):

2,4,4,6,8,8,8,10,12,12,14,16,16,16,16,18,20,20,22,24,24,24,
26,28,28,30,32,32,32,32,32,34,36,36,38,40,40,40,42,44,44,46,
48,48,48,48,50,52,52,54,56,56,56,58,60,60,62,64,64,64,64,64,64,66,...

($S_2(n)$ is the smallest integer such that $S_2(n)!$ is divisible by 2^n .)

Curious property: this is the sequence of even numbers, each number being repeated as many times as its exponent (of power 2) is.

This is one of irreductible functions, noted $S_2(k)$, which helps to calculate the Smarandache function.

Study this sequence.

UNSOLVED PROBLEM: 48

Primitive numbers (of power 3):

3,6,9,9,12,15,18,18,21,24,27,27,27,30,33,36,36,39,42,45,45,

48,51,54,54,54,57,60,63,63,66,69,72,72,75,78,81,81,81,81,84,

87,90,90,93,96,99,99,102,105,108,108,108,111,...

($S_3(n)$ is the smallest integer such that $S_3(n)!$ is divisible by 3^n .)

Curious property: this is the sequence of multiples of 3, each number being repeated as many times as its exponent (of power 3) is.

This is one of irreductible functions, noted $S_3(k)$, which helps to calculate the Smarandache function.

Study this sequence.

UNSOLVED PROBLEM: 49

Primitive numbers (of power p, p prime) {generalization}:

($S_p(n)$ is the smallest integer such that $S_p(n)!$ is divisible by p^n .)

Curious property: this is the sequence of multiples of p, each number being repeated as many times as its exponent (of power p) is.

These are the irreductible functions, noted $S_p(k)$, for any prime number p, which helps to calculate the Smarandache function.

Study this sequence.

UNSOLVED PROBLEM: 50

Let $a \in Q\setminus\{-1, 0, 1\}$. Solve the equation:

$$xa^{\frac{1}{x}} + \frac{1}{x} a^x = 2a.$$

[A generalization of the problem 0:123, Gazeta Matematică, No. 3/1980, p. 125.]

UNSOLVED PROBLEM: 51

(a) If (a, b) = 1, how many primes does the progression $ap_n + b$, n = 1, 2, ..., contain? where p_n is the n-th prime. But numbers of the form n! ? But n^n ?

(b) Same questions for $a^n + b$, $a \notin \{\pm 1, 0\}$.

(c) Same questions for $k^k + 1$ and $k^k - 1$, $k \in N^*$.

(Gamma 2/1986)

UNSOLVED PROBLEM: 52

(a) Let n be a non-null positive integer and d(n) the number of positive divisors of n. Of course, $d(n) \leq n$, and $d(n) = 1$ if and only if $n = 1$. For $n \geq 2$ we have $d(n) \geq 2$. Find the smallest k such that

$$\underbrace{d(d(\ldots d}_{k \text{ times}}(n)\ldots)) = 2$$

(b) Let $\sigma(n) = \sum_{\substack{d/n \\ d>o}} d$ and m a given positive integer

Find the smallest k such that

$$\underbrace{\sigma(\sigma(\ldots \sigma}_{k \text{ times}}(2)\ldots)) \geq m.$$

UNSOLVED PROBLEM: 53

Let $a_1, a_2, \ldots$ be a strictly increasing sequence of positive integers, and N(n) the number of terms of the sequence not greater than n.

(1) Find the smallest k such that

$$\underbrace{N(N(\ldots N}_{k \text{ times}}(n)\ldots)) \text{ is constant, for a given n.}$$

(2) If $a_1 \notin \{0,1\}$, find the smallest k such that

```
k     a          ≥ n, for a given n.
       a
  t     .
   i     .
    m     .
     e     a
      s     1
```

Particular Cases

(a) When $\{a_i\}_i$ is the sequence of primes.

(b) When $\{a_i\}_i$ is the sequence of m-th powers, for a given m : 0, 1, 2^m, 3^m, ...

For example, the sequence of perfect squares.

(c) $\{a_i\}_i$ is anyone of the well known sequences of positive integers, for example Fibonacci (or Lucas, Fermat, Bernoulli, Mersenne, van der Waerden, etc.) numbers.

UNSOLVED PROBLEM: 54

Let k be a non-zero integer. There are only a finite number of solutions in integers p, q, x, y, each greater than 1, of the equation $x^p - y^q = k$.

(On Catalan's conjecture) [For k = 1 this was conjectured by Cassels (1953) and proved by Tijdeman (1976).]

(Gamma 2/1986)

UNSOLVED PROBLEM: 55

Let $\{x_n\}_{n\geq 1}$ be a sequence of integers, and $o \leq k \leq q$ a digit. We define a <u>sequence of position</u>:

$$U_n^{(k)} = U^{(k)}(X_n) = \begin{cases} i, & \text{if } k \text{ is the } i\text{-th digit of } x_n; \\ o, & \text{in the other ones} \end{cases}$$

(For example: if $x_1 = 5$, $x_2 = 17$, $x_3 = 715$, ... and $K = 7$, then

$$U_1^{(7)} = U^{(7)}(x_1) = 0,\ U_2^{(7)} = 2,\ U_3^{(7)} = 1,\ \ldots)$$

(1) Study $\{U^{(k)}(p_n)\}_n$, where $\{p_n\}_n$ is the sequence of primes. Convergence, monotony.

The same questions for the sequences:

(2) $x_n = n!$, $n = 1, 2, \ldots$

(3) $x_n = n^n$, $n = 1, 2, \ldots$

Do the sequences of position $U_n^{(k)}$ contain an infinity of primes?

More generally: when $\{x_n\}_n$ is a sequence of rational numbers and $k \in N$.

UNSOLVED PROBLEM: 56

Let M be a number in a base b. All distinct digits of M are named generalized period of M. (For example, if M = = 104001144, its generalized period is g(M) = {0, 1, 4}.). Of course, $g(M) \subseteq \{0, 1, 2, \ldots, b - 1\}$.

The number of generalized period of M is equal to the number of groups of M such that each group contain all digits of g(M). (For example, $n_g(M) = 2$, $M = \underset{1}{104}\ \underset{2}{001144}$).

Length of the generalized period is equal to the number of its digits. (For example, $\ell_g(M) = 3$.)

Questions

(1) Find n_g, ℓ_g for p_n, $n!$, n^n, $\sqrt[n]{n}$.

(2) For a given $k \geq 1$, is there an infinity of primes p_n, or $n!$, or n^n, or $\sqrt[n]{n}$ which have a generalized period of length k? But which have the number of generalized periods equal to k?

(3) Let $a_1, a_2, \ldots, a_h$ distinct digits. Is there an infinity of primes p_n, or $n!$, or n^n, or $\sqrt[n]{n}$ which have as generalized period the set $\{a_1, a_2, \ldots, a_h\}$?

Remark

There exist arithmetic and geometrical progressions which contain an infinity of terms of given generalized period. For example, if g = {6, 7}, we construct an arithmetic progression: 67, 167, 267, ... and a geometrical one: 67, 670, 6700, ...

UNSOLVED PROBLEM: 57

Find the maximum r such that: the set {1, 2, ..., r} can be partitioned into n classes such that no class contains integers x, y, z with xy = z.

(On Schur's Problem)

Same question when $x^y = z$.

Same question when no integer can be the sum of another integer of its class. (A generalization of Schur's Problem.)

UNSOLVED PROBLEM: 58

Let N = {1, 2, ..., n}. Find the maxim number of elements extracted from N such that any m from these be not an arithmetic progression (n > m > 2).

Same question when the m elements must not be a geometrical progression.

More generally: Let R be a given m - ary relation on a field N. What is the maximum number of elements extracted from N such that any m from these be not in this relation? What happens when N has continuation power (for example when N is an interval from R)? [On van der Waerden's theorem.]

UNSOLVED PROBLEM: 59

Let ω numbers be $a + b\omega$, where ω is a complex n-th root of unity, $\omega^{n-1} + \omega^{n-2} + \ldots + 1 = 0$, which enjoy unique factorization. The units are: $\pm 1, \pm \omega, \pm \omega^2, \ldots, \pm \omega^{n-1}$.

Conjecture

The configuration of ω primes are symmetric of the 2n regular polygon.

(On Gaussian primes. A generalization of Einstein's integers.)

Reference

[1] R. K. Guy, Unsolved Problems in Number Theory, Springer-Verlag, New York, Heidelberg, Berlin, 1981, pp. 20-22, A16.

UNSOLVED PROBLEM: 60

The equation $x^3 + y^3 + z^3 = 1$ has as solutions (9, 10, -12) and (-6, -8, 9). How many other nontrivial integer solutions are there?

References

[1] V. L. Gardiner, R. B. Lazarus, and P. R. Stein, Solution of the Diophantine Equation $x^3 + y^3 = z^3 - d$, Math. Comput. 18 (1964) 408-413; MR 31 #119.

[2] J. C. P. Miller and M. F. C. Woollett, Solutions of the Diophantine Equation $x^3 + y^3 + z^3 = k$, J. London: Math. Soc. 30 (1955) 101-110; MR 16, 979.

[3] R. K. Guy. Unsolved Problems in Number Theory. Springer-Verlag, New York, Heidelberg, Berlin, 1981, p. 84, D5.

UNSOLVED PROBLEM: 61

Daniel Silverman asked if $\prod_{n=1}^{m} \frac{p_n+1}{p_n-1}$, where p_n is the n-th prime, is an integer for others $m \in \{1, 2, 3, 4, 8\}$. We conjecture that

$$R_m = \prod_{n=1}^{m} \frac{p_n+k}{p_n-k}, \text{ with } K \in N^*,$$

is an integer for a finite number of values of m. There is an infinite number of k for which no R_m is an integer.

Reference

[1] R. K. Guy, Unsolved Problems in Number Theory, Springer-Verlag, New York, Heidelberg, Berlin, 1981, p. 57, B48.

UNSOLVED PROBLEM: 62

ON A PROBLEM WITH INFINITE SEQUENCES

Let $1 \leq a_1 < a_2 < \ldots$ be an infinite sequence of integers such that any three members do not constitute an arithmetical progression. Is it true that always $\sum_{n\geq 1} 1/a_n \leq 2$? Is the function

$$S(\{a_n\}_{n\geq 1}) = \sum_{n\geq 1} 1/a_n$$

bijective? (biunivocal)

For example, $a_n = p^{n-1}$, $n \geq 1$, p is an integer > 1, has the property of the assumption, and $\sum_{n\geq 1} 1/a_n = 1 + \frac{1}{n-1} \leq 2$. Analogously for geometrical progressions.

More generally: let f be a function $f:R_+^m \to R_+^*$. We construct a sequence $0 < a_1 < a_2 < \ldots$ such that there is no $(a_{i_1}, \ldots, a_{i_m}, a_{i_{m+1}})$ with $f(a_{i_1}, \ldots, a_{i_m}) = a_{i_{m+1}}$. Find

$$\max_{\{a_n\}_{n\geq 1}} \Sigma_{1/a_n}.$$

(It's a generalization of a question from the problem E28, R. K. Guy, Unsolved Problems in Number Theory, Springer-Verlag, 1987, p. 127.)

Is the function

$$S(\{a_n\}\ n \geq 1)_{n\geq 1} = \sum 1/a_n$$

bijective?

UNSOLVED PROBLEM: 63

Square residues:

1,2,3,2,5,6,7,2,3,10,11,6,13,14,15,2,17,6,19,10,21,22,23,6,

5,26,3,14,29,30,31,2,33,34,35,6,37,38,39,10,41,42,43,22,15,

46,47,6,7,10,51,26,53,6,14,57,58,59,30,61,62,21,...

($s_r(n)$ is the largest square free number which divides n.)

Or, $s_r(n)$ is the number n released of its squares:

if $n = (p_1 \hat{} a_1) * \ldots * (p_r \hat{} a_r)$, with all p_i primes and all $a_i >=$

then $s_r(n) = p_1 * \ldots * p_r$.

Remark: at least the (2^2)*k-th numbers (k = 1, 2, 3, ...)

are released of their squares;

and more general: all (p^2)*k-th numbers (for all p prime,

and k = 1, 2, 3, ...) are released of their squares.

Study this sequence.

UNSOLVED PROBLEM: 64

Cubical residues:

1,2,3,4,5,6,7,4,9,10,11,12,13,14,15,4,17,18,19,20,21,22,23,

12,25,26,9,28,29,30,31,4,33,34,35,36,37,38,39,20,41,42,43,

44,45,46,47,12,49,50,51,52,53,18,55,28,...

($c_r(n)$ is the largest cube free number which divides n.)

Or, $c_r(n)$ is the number n released of its cubicals:

if $n = (p_1 \hat{} a_1) * \ldots * (p_r \hat{} a_r)$, with all p_i primes and all $a_i >=$

then $c_r(n) = (p_1 \hat{} b_1) * \ldots * (p_r \hat{} b_r)$, with all $b_i = \min\{2, a_i\}$.

Remark: at least the (2^3)*k-th numbers (k = 1, 2, 3, ...)

are released of their cubicals;

and more general: all (p^3)*k-th numers (for all p prime,

and k = 1, 2, 3, ...) are released of their cubicals.

Study this sequence.

UNSOLVED PROBLEM: 65

m-power residues (generalization):

($m_r(n)$ is the largest m-power free number which divides n.)

Or, $m_r(n)$ is the number n released of its m-powers:

if $n = (p_1 \hat{} a_1) * \ldots * (p_r \hat{} a_r)$, with all p_i primes and all $a_i >= 1$

then $m_r(n) = (p_1 \hat{} b_1) * \ldots * (p_r \hat{} b_r)$, with all $b_i = \min \{ m-1, a_i$

Remark: at least the (2^m)*k-th numbers (k = 1, 2, 3, ...) are released of their m-powers;

and more general: all (p^m)*k-th numers (for all p prime, and k = 1, 2, 3, ...) are released of their m-powers.

Study this sequence.

UNSOLVED PROBLEM: 66

Exponents (of power 2):

0,1,0,2,0,1,0,3,0,1,0,2,0,1,0,4,0,1,0,2,0,1,0,2,0,1,0,2,0,
1,0,5,0,1,0,2,0,1,0,3,0,1,0,2,0,1,0,3,0,1,0,2,0,1,0,3,0,1,
0,2,0,1,0,6,0,1,...

($e_2(n)$ is the largest exponent (of power 2) which divides n .)

Or, $e_2(n) = k$ if 2^k divides n but 2^(k+1) does not.

UNSOLVED PROBLEM: 67

Exponents (of power 3):

0,0,1,0,0,1,0,0,2,0,0,1,0,0,1,0,0,2,0,0,1,0,0,1,0,0,3,0,0,

1,0,0,1,0,0,2,0,0,1,0,0,1,0,0,2,0,0,1,0,0,1,0,0,2,0,0,1,0,

0,1,0,0,2,0,0,1,0,...

($e_3(n)$ is the largest exponent (of power 3) which divides n.)

Or, $e_3(n) = k$ if 3^k divides n but 3^(k+1) does not.

UNSOLVED PROBLEM: 68

Exponents (of power p) {generalization}:

($e_p(n)$ is the largest exponent (of power p) which divides n, where p is an integer >= 2 .)

Or, $e_p(n) = k$ if p^k divides n but p^(k+1) does not.

Study these sequences.

Reference:

Arizona State University, Hayden Library, "The Florentin Smarandache papers" special collection, Tempe, AZ 85287-1006, USA.

UNSOLVED PROBLEM: 69

Pseudo-primes:

2,3,5,7,11,13,14,16,17,19,20,23,29,30,31,32,34,35,37,38,41,
43,47,50,53,59,61,67,70,71,73,74,76,79,83,89,91,92,95,97,98,
101,103,104,106,107,109,110,112,113,115,118,119,121,124,125,
127,128,130,131,133,134,136,137,139,140,142,143,145,146, ...

(A number is pseudo-prime if some permutation of the digits is a prime number, including the identity permutation.)

(Of course, all primes are pseudo-primes, but not the reverse!)

Study this sequence.

UNSOLVED PROBLEM: 70

Pseudo-squares:

1,4,9,10,16,18,25,36,40,46,49,52,61,63,64,81,90,94,100,106,
108,112,121,136,144,148,160,163,169,180,184,196,205,211,225,
234,243,250,252,256,259,265,279,289,295,297,298,306,316,324,
342,360,361,400,406,409,414,418,423,432,441,448,460,478,481,
484,487,490,502,520,522,526,529,562,567,576,592,601,603,604,
610,613,619,625,630,631,640,652,657,667,675,676,691,729,748,
756,765,766,784,792,801,810,814,829,841,844,847,874,892,900,
904,916,925,927,928,940,952,961,972,982,1000, ...

(A number is a pseudo-square if some permutation of the digits is aperfect square, including the identity permutation.)

(Of course, all perfect squares are pseudo-squares, but not the reverse!)

One listed all pseudo-squares up to 1000.

Study this sequence.

UNSOLVED PROBLEM: 71

Pseudo-cubes:

1,8,10,27,46,64,72,80,100,125,126,152,162,207,215,216,251,
261,270,279,297,334,343,406,433,460,512,521,604,612,621,
640,702,720,729,792,800,927,972,1000,...

(A number is a pseudo-cube if some permutation of the digits is a cube, including the identity permutation.)

(Of course, all perfect cubes are pseudo-cubes, but not the reverse!)

One listed all pseudo-cubes up to 1000.

UNSOLVED PROBLEM: 72

Pseudo-m-powers:

(A number is a pseudo-m-power if some permutation of the digits is m-power, including the identity permutation; m >= 2.)

Study these sequences.

Reference:

Arizona State University, Hayden Library, "The Florentin Smarandache papers" special collection, Tempe, AZ 85287-1006, USA.

UNSOLVED PROBLEM: 73

Pseudo-factorials:

1,2,6,10,20,24,42,60,100,102,120,200,201,204,207,210,240,270,
402,420,600,702,720,1000,1002,1020,1200,2000,2001,2004,2007,
2010,2040,2070,2100,2400,2700,4002,4005,4020,4050,4200,4500,
5004,5040,5400,6000,7002,7020,7200,...

(A number is a pseudo-factorial if some permutation of the digits is a factorial number, including the identity permutation.)

(Of course, all factorials are pseudo-factorials, but not the reverse!)

One listed all pseudo-factorials up to 10000.

Procedure to obtain this sequence:

- calculate all factorials with one digit only (1!=1, 2!=2, and 3!=6), this is line_1 (of one digit pseudo-factorials): 1,2,6;
- add 0 (zero) at the end of each element of line_1, calculate all factorials with two digits (4!=24 only) and all permutations of their digits: this is line_2 (of two digits pseudo-factorials): 10,20,60; 24, 42;
- add 0 (zero) at the end of each element of line_2 as well as anywhere in between their digits, calculate all factorials with three digits (5!=120, and 6!=720) and all permutations of their digits:

this is line_3 (of three digits pseudo-factorials):

100,200,600,240,420,204,402; 120,720, 102,210,201,702,270,720;

and so on ...

to get from line_k to line_(k+1) do:

- add 0 (zero) at the end of each element of line_k as well as anywhere in between their digits,
  calculate all factorials with (k+1) digits
  and all permutations of their digits;

The set will be formed by all line_1 to the last line elements in an increasing order.

Study this sequence.

UNSOLVED PROBLEM: 74

Pseudo-divisors:

1,10,100,1,2,10,20,100,200,1,3,10,30,100,300,1,2,4,10,20,40,
100,200,400,1,5,10,50,100,500,1,2,3,6,10,20,30,60,100,200,
300,600,1,7,10,70,100,700,1,2,4,8,10,20,40,80,100,200,400,
800,1,3,9,10,30,90,100,300,900,1,2,5,10,20,50,100,200,500,1000,..

(The pseudo-divisors of n.)

(A number is a pseudo-divisor of n if some permutation of the digits is a divisor of n, including the identity permutation.)

(Of course, all divisors are pseudo-divisors,
but not the reverse!)

A strange property: any integer has an infinity of pseudo-divisors !!
because 10...0 becomes 0...01 = 1, by a circular permutation of its digits, and 1 divides any integer !

One listed all pseudo-divisors up to 1000 for the numbers 1, 2, 3, ..., 10.

Procedure to obtain this sequence:

- calculate all divisors with one digit only,
  this is line_1 (of one digit pseudo-divisors);
- add 0 (zero) at the end of each element of line_1,
  calculate all divisors with two digits
  and all permutations of their digits:
  this is line_2 (of two digits pseudo-divisors);
- add 0 (zero) at the end of each element of line_2 as well as anywhere in between their digits,
  calculate all divisors with three digits
  and all permutations of their digits:
  this is line_3 (of three digits pseudo-divisors);

and so on ...
to get from line_k to line_(k+1) do:

- add 0 (zero) at the end of each element of line_k as well

as anywhere in between their digits,
calculate all divisors with (k+1) digits
and all permutations of their digits;
The set will be formed by all line_1 to the last line elements in an increasing order.

Study this sequence.

UNSOLVED PROBLEM: 75

Pseudo-odd numbers:
1,3,5,7,9,10,11,12,13,14,15,16,17,18,19,21,23,25,27,29,30,
31,32,33,34,35,36,37,38,39,41,43,45,47,49,50,51,52,53,54,
55,56,57,58,59,61,63,65,67,69,70,71,72,73,74,75,76,...
(Some permutation of digits is an odd number.)

Study this sequence.

UNSOLVED PROBLEM: 76

Pseudo-triangular numbers:

1,3,6,10,12,15,19,21,28,30,36,45,54,55,60,61,63,66,78,82,

87,91,...

(Some permutation of digits is a triangular number.)

A triangular number has the general form: $n(n+1)/2$.

Study this sequence.

UNSOLVED PROBLEM: 77

Pseudo-even numbers:

0,2,4,6,8,10,12,14,16,18,20,21,22,23,24,25,26,27,28,29,30,

32,34,36,38,40,41,42,43,44,45,46,47,48,49,50,52,54,56,58,

60,61,62,63,64,65,66,67,68,69,70,72,74,76,78,80,81,82,83,

84,85,86,87,88,89,90,92,94,96,98,100,...

(The pseudo-even numbers.)

(A number is a pseudo-even number if some permutation of the digits is a even number, including the identity permutation.)

(Of course, all even numbers are pseudo-even numbers, but not the reverse!)

A strange property: an odd number can be a pseudo-even number!

One listed all pseudo-even numbers up to 100.

Study this sequence.

UNSOLVED PROBLEM: 78

Pseudo-multiples (of 5):

0,5,10,15,20,25,30,35,40,45,50,51,52,53,54,55,56,57,58,59,

60,65,70,75,80,85,90,95,100,101,102,103,104,105,106,107,

108,109,110,115,120,125,130,135,140,145,150,151,152,153,

154,155,156,157,158,159,160,165,...

(The pseudo-multiples of 5.)

(A number is a pseudo-multiple of 5 if some permutation of the digits is a multiple of 5, including the identity permutation.)

(Of course, all multiples of 5 are pseudo-multiples, but not the reverse!)

UNSOLVED PROBLEM: 79

Pseudo-multiples of p (p is an integer >= 2)

{generalizations}:

(The pseudo-multiples of p.)

(A number is a pseudo-multiple of p if some permutation of the digits is a multiple of p, including the identity permutation.)

(Of course, all multiples of p are pseudo-multiples, but not the reverse!)

Procedure to obtain this sequence:

- calculate all multiples of p with one digit only (if any),
  this is line_1 (of one digit pseudo-multiples of p);
- add 0 (zero) at the end of each element of line_1,
  calculate all multiples of p with two digits (if any)
  and all permutations of their digits:
  this is line_2 (of two digits pseudo-multiples of p);
- add 0 (zero) at the end of each element of line_2 as well
  as anywhere in between their digits,
  calculate all multiples with three digits (if any)
  and all permutations of their digits:
  this is line_3 (of three digits pseudo-multiples of p);

and so on ...

to get from line_k to line_(k+1) do:

- add 0 (zero) at the end of each element of line_k as well
  as anywhere in between their digits,
  calculate all multiples with (k+1) digits (if any)
  and all permutations of their digits;

The set will be formed by all line_1 to the last line elements in an increasing order.

Study this sequence.

UNSOLVED PROBLEM: 80

Square roots:

0,1,1,1,2,2,2,2,2,3,3,3,3,3,3,4,4,4,4,4,4,4,4,4,5,5,5,5,

5,5,5,5,5,5,6,6,6,6,6,6,6,6,6,6,6,6,7,7,7,7,7,7,7,7,7,

7,7,7,7,7,8,8,8,8,8,8,8,8,8,8,8,8,8,8,8,8,9,9,9,9,9,9,

9,9,9,9,9,9,9,9,9,9,9,9,10,10,10,10,10,10,10,10,10,10,

10,10,10,10,10,10,10,10,10,...

($s_q(n)$ is the superior integer part of square root of n.)

Remark: this sequence is the natural sequence, where each number is repeated 2n+1 times,

because between n^2 (included) and (n+1)^2 (excluded) there are (n+1)^2 - n^2 different numbers.

Study this sequence.

UNSOLVED PROBLEM: 81

Cubical roots:

0,1,1,1,1,1,1,1,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,2,3,3,3,

3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,3,

3,3,3,3,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,

4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,4,

4,4,4,4,4,...

($c_q(n)$ is the superior integer part of cubical root of n.)

Remark: this sequence is the natural sequence, where each number is repeated 3n^2 + 3n + 1 times,

because between n^3 (included) and (n+1)^3 (excluded) there are (n+1)^3 - n^3 different numbers.

UNSOLVED PROBLEM: 82

m-power roots:

($m_q(n)$ is the superior integer part of m-power root of n.)

Remark: this sequence is the natural sequence, where each number is repeated (n+1)^m - n^m times.

Study these sequences.

UNSOLVED PROBLEM: 83

Let $\sigma(n)$ be sum of divisors of n, $\Pi(x)$ number of primes not exceeding x, $\omega(n)$ number of distinct prime factors of n, $d(n)$ number of positive divisors of n, $p(n)$ the largest prime factor of n.

Let $f^{(k)}$ note $\underbrace{f \circ f \circ \ldots \circ f}_{k \text{ times}}$, for all function f.

Find the smallest k for which:

(a) for fixed n and m, we have $\sigma^{(k)}(n) > m$.

(b) for a fixed real x, with $x \geq 2$, we have $\Pi^{(k)}(x) = 1$.

(c) for a fixed n, we have $\omega^{(k)}(n) = 1$.

(d) for fixed n and m, we have $d^{(k)}(n) > m$.

(e) for a fixed n, we have

$$\underbrace{p(p(\ldots(p}_{k \text{ times}}(n)-1)\ldots)-1)-1 = 1.$$

Generalize for $\sigma_r(n)$, $\Pi(x; a, b)$, $\Omega(n)$.

Reference

[1] R. K. Guy, Unsolved Problems in Number Theory, Springer-Verlag, 1981; the problems: B2, B5, B8, B9; A17, E4; B11, B36, B38, B39, B12, B18, B32, B46.

UNSOLVED PROBLEM: 84

AN EXTENSION OF CARMICHAEL'S CONJECTURE

Conjecture

$\forall a \in N^*$, $\forall b \in N^*$, $\exists c \in N^*$: $\varphi(a) \cdot \varphi(b) = \varphi(c)$. For $a = 1$ it results Carmichael's conjecture. If this conjecture is true, then by mathematical induction it finds:

$$\forall a_1, \ldots, a_n \in N^*, \exists b \in N^*: \varphi(a_1) \ldots \varphi(a_n) = \varphi(b).$$

Reference

[1] R. K. Guy, Unsolved Problems in Number Theory, Springer-Verlag, 1981, p. 53, B39.

UNSOLVED PROBLEM: 85

ON CRITTENDEN AND VANDEN EYNDEN'S CONJECTURE

Is it possible to cover all (positive) integers with n geometrical progressions of integers?

*Find a necessary and sufficient condition for a general class of positive integer sequences such that, for a fixed n, there are n (distinct) sequences of this class which cover all integers.

Comment

(a) No. Let $a_1, \ldots, a_n$ be respectively the first term of each geometrical progression, and $q_1, \ldots, q_n$

respectively their rations. Let ρ be a prime, $\rho \notin \{a_1, \ldots, a_n, q_1, \ldots, q_n\}$. Then ρ does not belong to the union of these n geometrical progressions.

(b)* For example, the class of progressions:

$$A_f = \{\{a_n\}_{n\geq 1} : a_n = f(a_{n-1}, \ldots, a_{n-i}) \text{ for } n \geq i + 1, \text{ and}$$

$$i, a_1, \ldots, a_i, \in N^*\}$$

with the property:

$$\exists y \in N^*, \forall (x_1, \ldots, x_i) \in N^{*i} : f(x_1, \ldots, x_i) \neq y,$$

does it cover all integers?

But, if we change the property:

$$\forall y \in N^*, \exists (x_1, \ldots, x_i) \in N^{*i} : f(x_1, \ldots, x_i) = y?$$

(Generally no; see the geometrical progressions.)

This (solved and unsolved) problem remembers Crittenden and Vanden Eynden's conjecture.

Reference

[1] R. B. Crittenden and C. L. Vanden Eynden, Any n Arithmetic Progressions Covering the First 2^n Integers Covers All Integers, Proc. Amer. Math. Soc. 24 (1970) 475-481.

[2] R. B. Crittenden and C. L. Vanden Eynden, The Union of Arithmetic Progressions with Differences Not Less than k, Amer. Math. Monthly 79(1972) 630.

[3] R. K. Guy, Unsolved Problems in Number Theory, Springer-Verlag, New York, Heidelberg, Berlin, 1981, E23, 126.

UNSOLVED PROBLEM: 86

Consider the following equation:

$$(a - b \sqrt[m]{n})\ x + c \sqrt[m]{n} \cdot y + \sqrt[p]{q} \cdot z +$$

$$+ (d + e \cdot w) \sqrt[r]{s} = 0,$$

where a, b, c, d, e are constant integers; and the m-th, p-th and r-th roots are irrational distinct numbers. What conditions must the parameters m, n, p, q, r and s accomplish such that the equation admits integer solutions (x, y, z and w being variables)?

UNSOLVED PROBLEM: 87

Find the maximum number of interior points inside a parallelogram having an angle of $\pi/3$ such that the distance between any two points is greater or equal than 1. (The same question for a prism where all the faces are parallelograms with an angle of $\pi/3$.)

More generally: let $d > o$. Questions:

(a) Which is the maximum number of points included in a plane figure (a space body) such that the distance between any two points is greater or equal than d?

(b) Which is the minimum number of points $\{A_1, A_2, \ldots\}$ included in a plane figure (a space body) such that if another point A is added then there is an A_i with $|AA_i| < d$?

(c) Other variants of these questions if it considers: (1) only interior points; (2) interior and frontier points; (3) points on frontier only; (4) the distance is strictly greater than d; and (5) $|AA_i| \leq d$.

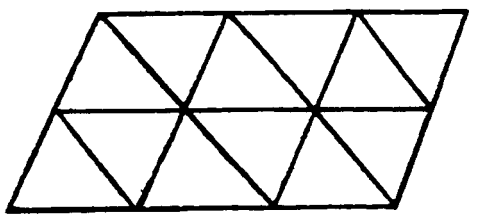
Solution

(I) We consider an equilateral triangle's network as indicated in the diagram displayed below, where ℓ_1 and ℓ_2 are the sides of our parallelogram. Clearly, this network gives the optimum construction of interior and frontier points keeping our property. It finds $(\lfloor \ell_1 \rfloor + 1)\ (\lfloor \ell_2 \rfloor + 1)$ points, where $\lfloor x \rfloor$ is the greatest integer less or equal than

x. If ℓ_1, $\ell_2 \in N^*$ we cannot take more than ℓ_1 interior points on a rule and ℓ_2 interior points on the other one, because it is not permitted to take points on the frontier.

If, for example, $\ell_1 \in N$, we can take $\lfloor \ell_1 \rfloor + 1$ interior points on a side. In conclusion: $n_{max} = \lceil \ell_1 \rceil \cdot \lceil \ell_2 \rceil$ interior points, where $\lceil x \rceil$ is the little integer greater or equal than x.

ℓ_1



ℓ_2

(II) For the prism of our problem, having the sides ℓ_1, ℓ_2, ℓ_3, of course, $n_{max} = \lceil \ell_1 \rceil \cdot \lceil \ell_2 \rceil \cdot \lceil \ell_3 \rceil$ [it results from (I) by considering the parallelograms (ℓ_1, ℓ_2) and (ℓ_2, ℓ_3)].

(III) These are generally open (unsolved) questions. For particular cases see [1].

Reference:

[1] Smarandache, Florentin, Problèmes avec et sans ... problèmes! (problems 5.43 [p. 67], respectively 5.44 [p. 62]), Somipress, Fès, Morocco, 1983 (M. R.: 84R: 00003).

UNSOLVED PROBLEM: 88

*Find all real solutions of the equation $x^y - \lfloor x \rfloor = y$, where $\lfloor x \rfloor$ is the greatest integer less or equal than x.

Solution: It is a generalization of a Putnam competition: $(x^3 - \lfloor x \rfloor = 3)$.

(1) When $y \in R \setminus Q$ the author is not able to answer.

(2) If $y = 0$ then $x \in [1, 2)$.

(3) If $y = 1$, $\exists\, x \in R$.

(4) If y is an odd integer > 1, then $x = \sqrt[y]{y+1} \in (1, 2)$, is the unique solution of our equation:

Let us use the functions

$f(x) = x^n - n$, $n \in N$, $n > 1$, where n is an odd integer,

$g(x) = \lfloor x \rfloor$, $f, g: R \to R$, and their graphics:

Fig. 2

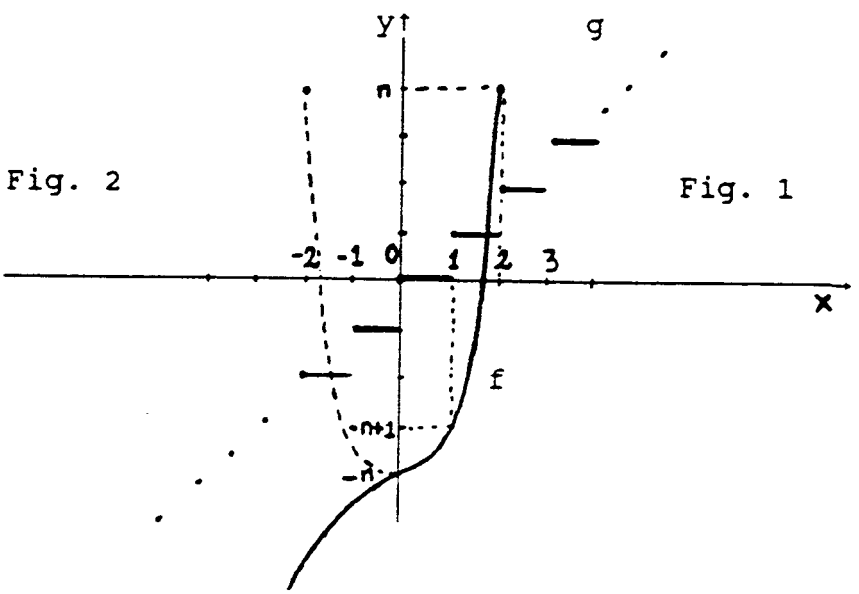


Fig. 1

(5) If $y = 2$, then $x_1 = \sqrt[2]{3}$ and $x_2 = -1$.

(6) If $y \in N$ and y is an even integer ≥ 4, then

$x_1 = \sqrt[y]{y+1}$, $x_2 = -\sqrt[y]{y-2}$. (It is sufficient to observe that $f(x) = x^n - n$, where n is an even integer ≥ 4, is an even function; we consider $f: R \to R$ (Fig. 2).).

(7) If $y = -1$ the $x = -1/2$, because it results in $1/x \in Z^*$, hence $x = 1/k$ with $k \in Z^*$, whence $k = -2$.

(8) If $y \in Z$, $y < -1$, $\exists\, x \in R$, because it results $1/x^m \in Z^*$, whence $x^m = 1/k$, $k \in Z^*$, where $m = -y > 0$. For an even m, we have $x = \pm \sqrt[m]{1/k}$ with $k \geq 1$, but $\lfloor \pm \sqrt[m]{1/k} \rfloor \in \{0, \pm 1\}$ and $k + m \geq 2$; hence $\exists\, x \in R$. For an odd m, we have $x = \sqrt[m]{1/k}$ with $k \in Z^*$; hence $k - \lfloor \sqrt[m]{1/k} \rfloor = -m$ whence $k = -m - 1 < 0$ and $x = -\sqrt[m]{1/(m+1)} = -(1 - y)^{1/y}$.

(9) Let $y = 1/n$, $n \in N$, n is odd ≥ 3, then $x_1 = (1/n)^n$, $x_2 = \left(\frac{1-n}{n}\right)^n$. Because the function $f(x) = x^{\frac{1}{n}} - \frac{1}{n}$, $n \in N$, n is odd ≥ 3, $f: R \to R$, has the following graphic:

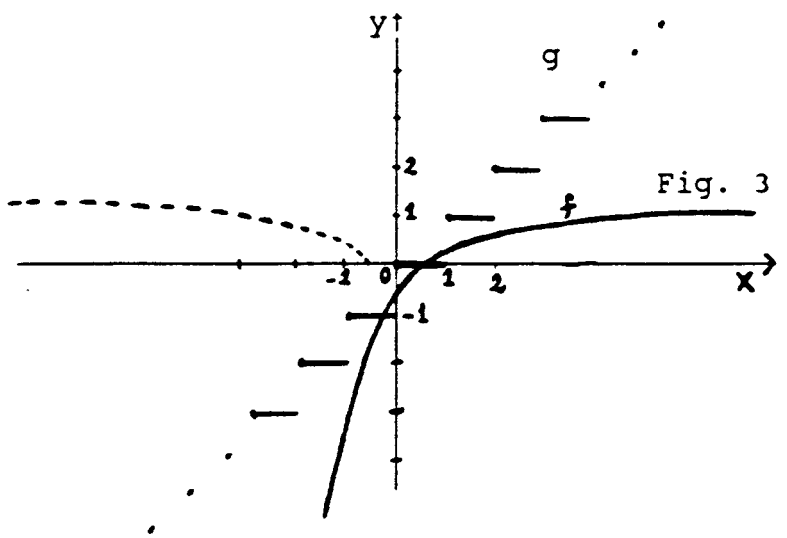


Fig. 3

For $x \geq 2$, $h(x) = x^{\frac{1}{n}} - x + 1 - \frac{1}{n} < 0$ (because $h'(x) <$ < 0 when $x \geq 2$, hence $h(x) \leq h(2) < 0$ for $x \geq 2$). There exists an unique positive real solution.

For $x \leq -1$, $k(x) = x^{\frac{1}{n}} - \frac{1}{n} - x > 0$; there exists an unique negative real solution.

(10) If $y = 1/n$, $n \in N$, n is even ≥ 2, then $x \geq 0$ involves $x = (1/n)^n$, because f is an even function (Fig. 4).

(11) If $y = -1/n$, $n \in N$, $n \geq 2$.

$$1/x^{1/n} + 1/n = \lfloor x \rfloor .$$

Whence $x^{1/n} \in Q$, it results $x = a^n$, $a \in Q$. Hence $1/a + 1/n \in Z$, whence $1/a = k - 1/n$, $k \in Z$, thus $a = \frac{n}{kn-1}$ or $k = \left\lfloor \left(\frac{n}{kn-1} \right)^n \right\rfloor$. Of course $k \neq 0$. If $k = 1$, we have $\left(1 + \frac{1}{n-1}\right)^n > 2$. If $k \geq 2$, we have $\left(\frac{n}{kn-1}\right)^n < 1$.

(a) If n is odd then $k = -1$, whence $x = \left(-\frac{n}{n+1}\right)^n$ is a solution.

(b) If n is even, there is no $x \in R$. For $k \leq -2$, we have $\left(\frac{n}{kn-1}\right)^n > -1$.

(12) $y = \frac{m}{n} \in Q \backslash Z$, $m \neq 1$.

* The author is not able to solve the equation in this case.

UNSOLVED PROBLEM: 89

Prove that on a circular disk of radius r there are least n points such that the distance between any two is greater or equal than d, where:

$$n = \sum_{k=0}^{\left[\frac{r}{d}\right]} [\pi/\arcsin\, d/(2(r-kd))] \text{ if } \frac{r}{d} \notin N,$$

or

$$n = 1 + \sum_{k=0}^{\frac{n}{d}-1} [\pi/\arcsin\, d/(2(r-kd))] \text{ if } \frac{r}{d} \in N.$$

Is n equal to the maximum number of points on the disk with this property?*

Generalize for an arbitrary figure in plane.**

Generalize also for an arbitrary corps in space.**

Proof

(a) Let $\zeta_d(r)$ be the maximum number of points on the circumference of a circle of radius r such that the distance between any two is greater or equal than d. Hence, the cord which unites two points from these is $\geq$ d (see the picture 1). We take it precisely d.

$$\frac{d}{\sin x} = \frac{r}{\sin\left(\frac{\pi}{2} - x\right)} \text{ whence } \sin\frac{x}{2} = \frac{d}{2r} \text{ hence}$$

$$x = 2 \arcsin \frac{d}{2r} \quad \text{(in radians)}.$$ We divide 2π to x

and it results in:

$$\zeta_d(r) = \left[\pi / \arcsin \frac{d}{2r}\right].$$

We proceed analogously with the circle C_1 of radius r-d, concentric with the first C_0, obtaining ζ_d (r-d), etc. This method ends at the step $k = \left[\frac{r}{d}\right]$ for which $0 < r-kd < d$.

When $\frac{r}{d}$ is an integer, the last drawn circle will be a point, precisely the center of these circles.

This construction mode (with point networks lying on concentric circles such that between two some circles the distance is equal to d) ensures the distance condition of all points.

It remarks that if since start $d > 2r$ the arcsin there is not, therefore our problem is impossible (there is no point). And, if $d = 0$, we obtain an infinite of points.

(b) *This construction is close to an optimum one (in the author's conception). But the author cannot prove if this is or is not optimum. There are many constructions of point networks [on squares, on (equilatera) triangles,

etc.]. For which point networks is n maximum? (Here it is an open question).

In our problem, when $\frac{r}{d}$ is very great perhaps the following <u>construction</u> is more advantageous (see picture 2):

We take a point P_1 on the circumference F_0 of our disk Z_0. With a compass we draw a circle arc A_1 (of radius d, having the center in P_1) which cuts F_0 in P_2 (to right). Afterwards we again draw a circle arc A_2 (of radius d, having the center in P_2) which cuts F_0 in P_3 (to right), etc. On F_0 we find ζ_0 points.

We can still take another points in the hachured zone Z_1 only. We construct these points on the frontier F_1 of Z_1, analogous: with a compass of radius d, of center R_1 at the start, etc. ($R_1 = A_1 \cap A_t$, where A_t is the last circle arc). On F_1 we find ζ_1 points.

This method ends when $Z_k = \emptyset$. It obtains at least n points having the property of our assumption. But, does it obtain more points than n?

(c) **These are two general open questions. For particular cases see [1].

Reference:

[1] Smarandache, Florentin, "Problèmes avec et sans ... problèmes!," the problems 5.43 (p. 61), respectively 5.44 (p. 62); Somipress, Fès, Morocco, 1983 (see MR:84K:00003).

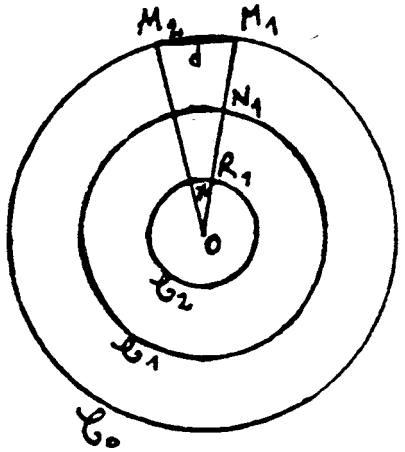


Fig. 1

$OM_1 = r$

$M_1M_2 = d$

$\sphericalangle M_1OM_2 = x$

$M_1N_1 = N_1R_1 = d$

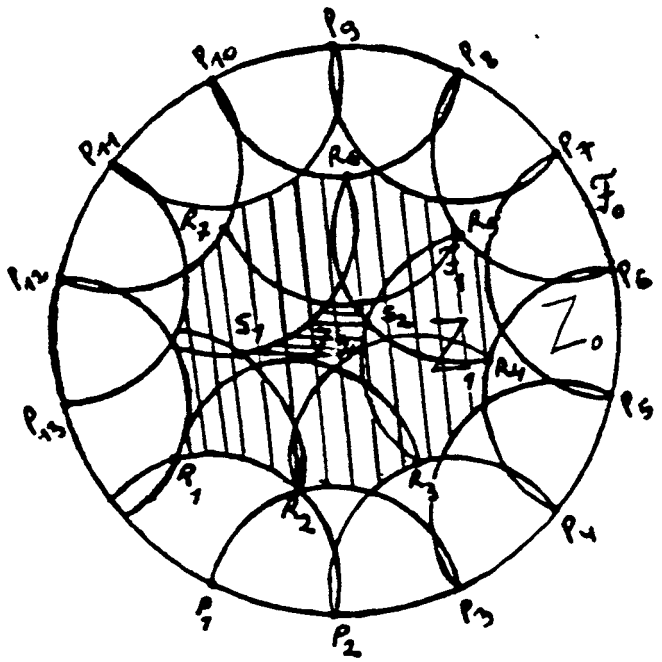


Fig. 2

UNSOLVED PROBLEM: 90

Prime base:

0,1,10,100,101,1000,1001,10000,10001,10010,10100,100000,

100001,1000000,1000001,1000010,1000100,10000000,10000001,

100000000,100000001,100000010,100000100,1000000000,1000000001,

1000000010,1000000100,1000000101,...

(Each number n written in the prime base.)

(I define over the set of natural numbers the following infinite base: $p_0 = 1$, and for $k >= 1$ p_k is the k-th prime number.)

Every positive integer A may be uniquely written in the prime base as:

$$A = (\overline{a_n \ldots a_1 a_0}_{(SP)}) \quad \stackrel{def}{===} \quad \sum_{i=0}^{n} a_i p_i \text{ , with all } a_i = 0 \text{ or } 1, \text{ (of course } a_n = 1),$$

in the following way:

- if $p_n <= A < p_{n+1}$ then $A = p_n + r_1$;
- if $p_m <= r_1 < p_{m+1}$ then $r_1 = p_m + r_2$, $m < n$;

and so on untill one obtains a rest $r_j = 0$.

Therefore, any number may be written as a sum of prime numbers + e, where e = 0 or 1.

If we note by p(A) the superior part of A (i.e. the largest prime less than or equal to A), then

A is written in the prime base as:

$$A = p(A) + p(A-p(A)) + p(A-p(A)-p(A-p(A))) + \ldots .$$

This base is important for partitions with primes.

Study this sequence.

UNSOLVED PROBLEM: 91

Square base:

0,1,2,3,10,11,12,13,20,100,101,102,103,110,111,112,1000,1001,

1002,1003,1010,1011,1012,1013,1020,10000,10001,10002,10003,

10010,10011,10012,10013,10020,10100,10101,100000,100001,100002,

100003,100010,100011,100012,100013,100020,100100,100101,100102,

100103,100110,100111,100112,101000,101001,101002,101003,101010,

101011,101012,101013,101020,101100,101101,101102,1000000,...

(Each number n written in the square base.)

(I define over the set of natural numbers the following infinite base: for $k >= 0$ $s_k = k^2$.)

Every positive integer A may be uniquely written in the square base as:

$$A = (\overline{a_n \ldots a_1 a_0})_{(S2)} \quad \stackrel{def}{===} \quad \sum_{i=0}^{n} a_i s_i \text{ , with } a_i = 0 \text{ or } 1 \text{ for } i >= 2,$$

$0 <= a_0 <= 3$, $0 <= a_1 <= 2$, and of course $a_n = 1$, in the following way:

- if $s_n <= A < s_{n+1}$ then $A = s_n + r_1$;
- if $s_m <= r_1 < p_{m+1}$ then $r_1 = s_m + r_2$, $m < n$;

and so on untill one obtains a rest $r_j = 0$.

Therefore, any number may be written as a sum of squares (1 not counted as a square -- being obvious) + e, where e = 0, 1, or 3.

If we note by s(A) the superior square part of A (i.e. the largest square less than or equal to A), then A is written in the square base as:

$$A = s(A) + s(A-s(A)) + s(A-s(A)-s(A-s(A))) + \ldots .$$

This base is important for partitions with squares.

Study this sequence.

UNSOLVED PROBLEM: 92

m-power base (generalization):

(Each number n written in the m-power base, where m is an integer >= 2.)

(I define over the set of natural numbers the following infinite m-power base: for $k >= 0$ $t_k = k^m$.)

Every positive integer A may be uniquely written in the m-power base as:

$$A = (\overline{a_n \ldots a_1 a_0}_{(SM)}) \quad \stackrel{def}{===} \quad \sum_{i=0}^{n} a_i t_i \text{ , with } a_i = 0 \text{ or } 1 \text{ for } i >= m,$$

$$0 <= a_i <= \left| \frac{((i+2)^m - 1)}{(i+1)^m} \right| \text{ (integer part)}$$

for $i = 0, 1, \ldots, m-1$, $a_i = 0$ or 1 for $i >= m$, and of course a_n = 1

in the following way:

- if $t_n <= A < t_{n+1}$ then $A = t_n + r_1$;
- if $t_m <= r_1 < t_{m+1}$ then $r_1 = t_m + r_2$, $m < n$;

and so on untill one obtains a rest $r_j = 0$.

Therefore, any number may be written as a sum of m-powers (1 not counted as an m-power -- being obvious) + e, where e = 0, 1, 2, ..., or 2^m-1.

If we note by t(A) the superior m-power part of A (i.e. the largest m-power less than or equal to A), then A is written in the m-power base as:

$$A = t(A) + t(A-t(A)) + t(A-t(A)-t(A-t(A))) + \ldots$$

This base is important for partitions with m-powers.

Study this sequence.

UNSOLVED PROBLEM: 93

Generalized base:

(Each number n written in the generalized base.)

(I define over the set of natural numbers the following infinite generalized base: $1 = g_0 < g_1 < \ldots < g_k < \ldots$.)

Every positive integer A may be uniquely written in the generalized base as:

$$A = \overline{(a_n \ldots a_1 a_0)}_{(SG)} \stackrel{def}{===} \sum_{i=0}^{n} a_i g_i \text{ , with } 0 <= a_i <= \lfloor (g_{i+1} - 1) / g_i \rfloor$$

(integer part) for i = 0, 1, ..., n, and of course $a_n >= 1$, in the following way:

- if $g_n <= A < g_{n+1}$ then $A = g_n + r_1$;
- if $g_m <= r_1 < g_{m+1}$ then $r_1 = g_m + r_2$, $m < n$;

and so on untill one obtains a rest $r_j = 0$.

If we note by g(A) the superior generalized part of A (i.e. the largest g_i less than or equal to A), then A is written in the m-power base as:

$$A = g(A) + g(A-g(A)) + g(A-g(A)-g(A-g(A))) + \ldots$$

This base is important for partitions: the generalized base may be any infinite integer set (primes, squares, cubes, any m-powers, Fibonacci/Lucas numbers, Bernoully numbers, etc.) those partitions are studied.

A particular case is when the base verifies: $2g_i >= g_{i+1}$ for any i, and $g_0 = 1$, because all coefficients of a written number in this base will be 0 or 1.

Study this sequence.

UNSOLVED PROBLEM: 94

Odd sieve:

7,13,19,23,25,31,33,37,43,47,49,53,55,61,63,67,73,75,79,83,
85,91,93,97,...

(All odd numbers that are not equal to the difference of two primes.)

A sieve is used to get this sequence:

- substract 2 from all prime numbers and obtain a temporary sequence;
- choose all odd numbers that do not belong to the temporary one.

Study this sequence.

UNSOLVED PROBLEM: 95

Binary sieve:

1,3,5,9,11,13,17,21,25,27,29,33,35,37,43,49,51,53,57,59,65,
67,69,73,75,77,81,85,89,91,97,101,107,109,113,115,117,121,
123,129,131,133,137,139,145,149,...

(Starting to count on the natural numbers set at any step from 1:

- delete every 2-nd numbers
- delete, from the remaining ones, every 4-th numbers

... and so on: delete, from the remaining ones, every (2^k)-th numbers, k = 1, 2, 3, ...)

Conjectures:

- there are an infinity of primes that belong to this sequence;
- there are an infinity of numbers of this sequence which are not prime.

UNSOLVED PROBLEM: 96

Trinary sieve:

1,2,4,5,7,8,10,11,14,16,17,19,20,22,23,25,28,29,31,32,34,35,
37,38,41,43,46,47,49,50,52,55,56,58,59,61,62,64,65,68,70,71,
73,74,76,77,79,82,83,85,86,88,91,92,95,97,98,100,101,103,104,
106,109,110,112,113,115,116,118,119,122,124,125,127,128,130,
131,133,137,139,142,143,145,146,149,...

(Starting to count on the natural numbers set at any step from 1:

- delete every 3-rd numbers
- delete, from the remaining ones, every 9-th numbers

... and so on: delete, from the remaining ones, every (3^k)-th numbers, k = 1, 2, 3, ...)

Conjectures:

- there are an infinity of primes that belong to this sequence;
- there are an infinity of numbers of this sequence which are not prime.

UNSOLVED PROBLEM: 97

n-ary sieve (generalization, n >= 2):

(Starting to count on the natural numbers set at any step from 1:

- delete every n-th numbers;
- delete, from the remaining ones, every (n^2)-th numbers;

... and so on: delete, from the remaining ones, every (n^k)-th numbers, k = 1, 2, 3, ...)

Conjectures:

- there are an infinity of primes that belong to this sequence;
- there are an infinity of numbers of this sequence which are not prime.

UNSOLVED PROBLEM: 98

Consecutive sieve:

1,3,5,9,11,17,21,29,33,41,47,57,59,77,81,101,107,117,131,149.
153,173,191,209,213,239,257,273,281,321,329,359,371,401,417,
441,435,491,...

(From the natural numbers set:

- keep the first number,
  delete one number out of 2 from all remaining numbers;
- keep the first remaining number,
  delete one number out of 3 from the next remaining numbers;
- keep the first remaining number,
  delete one number out of 4 from the next remaining numbers;

... and so on, for step k (k >= 2):

- keep the first remaining number,
  delete one number out of k from the next remaining numbers;

... .)

This sequence is much less dense than the prime number sequence,
and their ratio tends to $p_n : n$ as n tends to infinity.
For this sequence we choosed to keep the first remaining
number at all steps,
but in a more general case:
the kept number may be any among the remaining k-plet
(even at random).

Study this sequence.

UNSOLVED PROBLEM: 99

General-sequence sieve:

Let $u_i > 1$, for i = 1, 2, 3, ..., a stricly increasing positive integer sequence. Then:

From the natural numbers set:

- keep one number among 1, 2, 3, ..., $u_1 - 1$,
  and delete every u_1-th numbers;
- keep one number among the next $u_2 - 1$ remaining numbers,
  and delete every u_2-th numbers;

... and so on, for step k (k >= 1):

- keep one number among the next $u_k - 1$ remaining numbers,
  and delete every u_k-th numbers;

... .

Problem: study the relationship between sequence u_i, i = 1, 2, 3, ..., and the remaining sequence resulted from the general sieve.

u_i, previously defined, is called sieve generator.

Study this sequence.

UNSOLVED PROBLEM: 100

General residual sequence:

$(x + C_1)...(x + C_{F(m)})$, m = 2, 3, 4, ...,

where C_i, 1 <= i <= F(m), forms a reduced set of residues mod m,

x is an integer, and F is Euler's totient.

The General Residual Sequence is induced from the

The Residual Function (see <Libertas Mathematica>):

Let L : ZxZ --> Z be a function defined by

$$L(x,m)=(x + C_1)...(x + C_{F(m)}),$$

where C_i, 1 <= i <= F(m), forms a reduced set of residues mod m,

m >= 2, x is an integer, and F is Euler's totient.

The Residual Function is important because it generalizes the classical theorems by Wilson, Fermat, Euler, Wilson, Gauss, Lagrange, Leibnitz, Moser, and Sierpinski all together.

For x=0 it's obtained the following sequence:

$L(m) = C_1 \ldots C_{F(m)}$, where m = 2, 3, 4, ...

(the product of all residues of a reduced set mod m):

1,2,3,24,5,720,105,2240,189,3628800,385,479001600,19305,

896896,2027025,20922789888000,85085,6402373705728000,

8729721,47297536000,1249937325,...

which is found in "The Handbook of Integer Sequences", by N. J. A. Sloane, Academic Press, USA, 1973.

The Residual Function extends it.

Study this sequence.

References:

Fl. Smarandache, "A numerical function in the congruence theory", in <Libertah Mathematica>, Texas State University, Arlington, 12, pp. 181-185, 1992;
see <Mathematical Reviews> 93i:11005 (11A07), p.4727, and <Zentralblatt fur Mathematik>, Band 773(1993/23), 11004 (11A);

Fl. Smarandache, "Collected Papers" (Vol. 1), Ed. Tempus, Bucharest, 1994 (to appear);

Arizona State University, Hayden Library, "The Florentin Smarandache papers" special collection, Tempe, AZ 85287-1006, USA.

Student Conference, University of Craiova, Department of Mathematics, April 1979, "Some problems in number theory" by Florentin Smarandache.

UNSOLVED PROBLEM: 101

Code puzzle:

151405,202315,2008180505,06152118,06092205,190924,1905220514,

0509070820,14091405,200514,051205220514,...

Using the following letter-to-number code:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

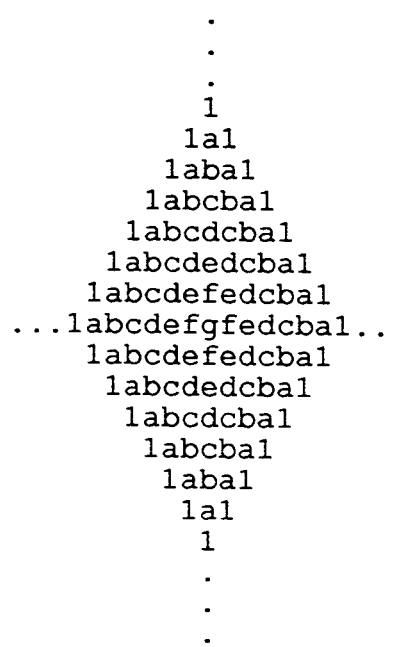
then $c_p(n)$ = the numerical code of the spelling of n in English language; for exemple: 1 = ONE = 151405, etc.

Find a better codification (one sign only for each letter).

UNSOLVED PROBLEM: 102

Numerical carpet:

has the general form



On the border of level 0, the elements are equal to "1";

they form a rhomb.

Next, on the border of level 1, the elements are equal to "a",

where "a" is the sum of all elements of the previous border;

the "a"s form a rhomb too inside the previous one.

Next again, on the border of level 2, the elements are equal to "b",

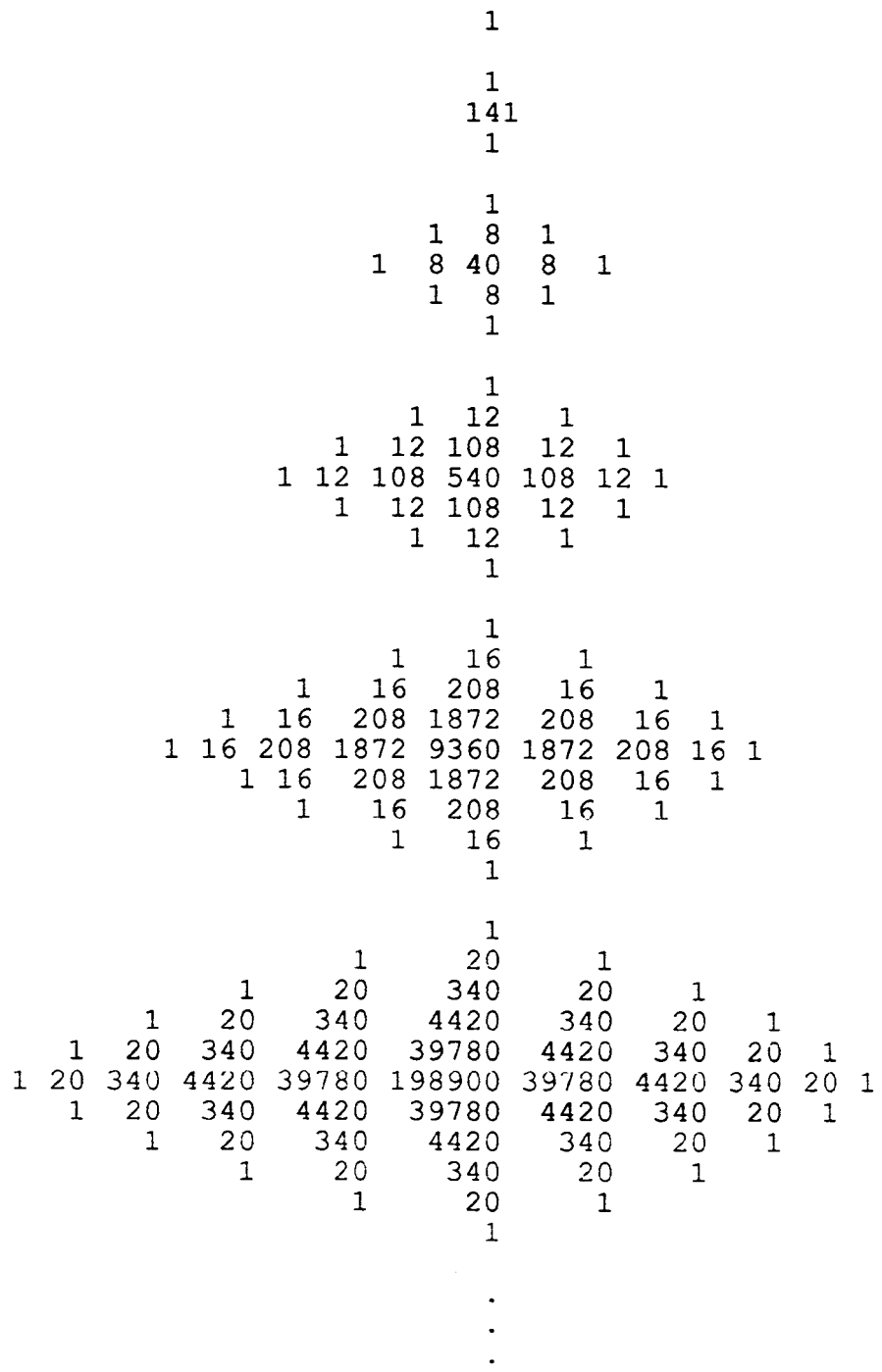
where "b" is the sum of all elements of the previous border;

the "b"s form a rhomb too inside the previous one.

And so on...

The numerical carpet is symmetric and esthetic, in its middle g is the sum of all carpet numbers (the core).

Look at a few terms of the Numerical Carpet:



Or, under other form:

```
1
1  4
1  8  40
1 12 108   504
1 16 208  1872   9360
1 20 340  4420  39780  198900
1 24 504  8568 111384 1002456   5012280
1 28 700 14700 249900 3248700  29238300 146191500
1 32 928 23200 487200 8282400 107671200 969040800 4845204000
.....................................................................
.
.
.
```

General Formula:

$$C(n,k) = 4n \left| \prod_{i=1}^{k} \right| (4n-4i+1) \text{ for } 1 <= k <= n,$$

and $C(n,0) = 1$.

Study this multi-sequence.

References:

Arizona State University, Hayden Library, "The Florentin Smarandache papers" special collection, Tempe, AZ 85287-1006, USA.

Student Conference, University of Craiova, Department of Mathematics, April 1979, "Some problems in number theory" by Florentin Smarandache.

Fl. Smarandache, "Collected Papers" (Vol. 1), Ed. Tempus, Bucharest, 1994 (to appear);

UNSOLVED PROBLEM: 103

Table:

6,10,14,18,26,30,38,42,42,54,62,74,74,90,...

(t(n) is the largest even number such that any other even number not exceeding it is the sum of two of the first n odd primes.)

It helps to better understand Goldbach's conjecture:

- if t(n) is unlimited, then the conjecture is true;
- if t(n) is constant after a certain rank, then the conjecture is false.

Also, the table gives how many times an even number is written as a sum of two odd primes, and in what combinations.

Of course, $t(n) <= 2p_n$, where p_n is the n-th odd prime, n = 1, 2, 3, ...

Here is the table:

+	3	5	7	11	13	17	19	23	29	31	37	41	43	47
3	6	8	10	14	16	20	22	26	32	34	40	44	46	50
5		10	12	16	18	22	24	28	34	36	42	46	48	52
7			14	18	20	24	26	30	36	38	44	48	50	54
11				22	24	28	30	34	40	42	48	52	54	58
13					26	30	32	36	42	44	50	54	56	60
17						34	36	40	46	48	54	58	60	64
19							38	42	48	50	56	60	62	66
23								46	52	54	60	64	66	70
29									58	60	66	70	72	76
31										62	68	72	74	78
37											74	78	80	84
41												82	84	88
43													86	90
47														94

Study this table and table sequence.

UNSOLVED PROBLEM: 104

Second table:

9,15,21,29,39,47,57,65,71,93,99,115,129,137,...

(v(n) is the largest odd number such that any odd number >= 9 not exceeding it is the sum of three of the first n odd primes.)

It helps to better understand Goldbach's conjecture for three primes:

- if v(n) is unlimited, then the conjecture is true;
- if v(n) is constant after a certain rank, then the conjecture is false.

(Vinogradov proved in 1937 that any odd number greater than 3^(3^15) satisfies this conjecture.
But what about values less than 3^(3^15) ?)

Also, the table gives you in how many different combinations an odd number is written as a sum of three odd primes, and in what combinations.

Of course, $v(n) <= 3p_n$, where p_n is the n-th odd prime, n = 1, 2, 3, ... It is also generalized for the sum of m primes, and how many times a number is written as a sum of m primes (m > 2).

This is a 3-dimensional 14x14x14 table, that we can expose only as 14 planar 14x14 tables (using the previous table):

3 +	3	5	7	11	13	17	19	23	29	31	37	41	43	47	
3	9	11	13	17	19	23	25	29	35	37	43	47	49	53	. . .
5		13	15	19	21	25	27	31	37	39	45	49	51	55	.
7			17	21	23	27	29	33	39	41	47	51	53	57	.
11				25	27	31	33	37	43	45	51	55	57	61	.
13					29	33	35	39	45	47	53	57	59	63	.
17						37	39	43	49	51	57	61	63	67	.
19							41	45	51	53	59	63	65	69	.
23								49	55	57	63	67	69	73	.
29									61	63	69	73	75	79	.
31										65	71	75	77	81	.
37											77	81	83	87	.
41												85	87	91	.
43													89	93	.
47														97	.
.	.														.
.	.														.
.	.														.

5 +	3	5	7	11	13	17	19	23	29	31	37	41	43	47	
3	11	13	15	19	21	25	27	31	37	39	45	49	51	55	. . .
5		15	17	21	23	27	29	33	39	41	47	51	53	57	.
7			19	23	25	29	31	35	41	43	49	53	55	59	.
11				27	29	33	35	39	45	47	53	57	59	63	.
13					31	35	37	41	47	49	55	59	61	65	.
17						39	41	45	51	53	59	63	65	69	.
19							43	47	53	55	61	65	67	71	.
23								51	57	59	65	69	71	75	.
29									63	65	71	75	77	81	.
31										67	73	77	79	83	.
37											79	83	85	89	.
41												87	89	93	.
43													91	95	.
47														99	.
.	.														.
.	.														.
.	.														.

7 +	3	5	7	11	13	17	19	23	29	31	37	41	43	47	
3	13	15	17	21	23	27	29	33	39	41	47	51	53	57	. . .
5		17	19	23	25	29	31	35	41	43	49	53	55	59	.
7			21	25	27	31	33	37	43	45	51	55	57	61	.
11				29	31	35	37	41	47	49	55	59	61	65	.
13					33	37	39	43	49	51	57	61	63	67	.
17						41	43	47	53	55	61	65	67	71	.
19							45	49	55	57	63	67	69	73	.
23								53	59	61	67	71	73	77	.
29									65	67	73	77	79	83	.
31										69	75	79	81	85	.
37											81	85	87	91	.
41												89	91	95	.
43													93	97	.
47														101	.
	. . . . .														
	.														.
	.														.
	.														.

11 +	3	5	7	11	13	17	19	23	29	31	37	41	43	47	
3	17	19	21	25	27	31	33	37	43	45	51	55	57	61	. . .
5		21	23	27	29	33	35	39	45	47	53	57	59	63	.
7			25	29	31	35	37	41	47	49	55	59	61	65	.
11				33	35	39	41	45	51	53	59	63	65	69	.
13					37	41	43	47	53	55	61	65	67	71	.
17						45	47	51	57	59	65	69	71	75	.
19							49	53	59	61	67	71	73	77	.
23								57	63	65	71	75	77	81	.
29									69	71	77	81	83	87	.
31										73	79	83	85	89	.
37											85	89	91	95	.
41												93	95	99	.
43													97	101	.
47														105	.
	. . . . .														
	.														.
	.														.
	.														.

13 +	3	5	7	11	13	17	19	23	29	31	37	41	43	47	
3	19	21	23	27	29	33	35	39	45	47	53	57	59	63	.
5		23	25	29	31	35	37	41	47	49	55	59	61	65	.
7			27	31	33	37	39	43	49	51	57	61	63	67	.
11				35	37	41	43	47	53	55	61	65	67	71	.
13					39	43	45	49	55	57	63	67	69	73	.
17						47	49	53	59	61	67	71	73	77	.
19							51	55	61	63	69	73	75	79	.
23								59	65	67	73	77	79	83	.
29									71	73	79	83	85	89	.
31										75	81	85	87	91	.
37											87	91	93	97	.
41												95	97	101	.
43													99	103	.
47														107	.

. . .

. .

. .

17 +	3	5	7	11	13	17	19	23	29	31	37	41	43	47	
3	23	25	27	31	33	37	39	43	49	51	57	61	63	67	.
5		27	29	33	35	39	41	45	51	53	59	63	65	69	.
7			31	35	37	41	43	47	53	55	61	65	67	71	.
11				39	41	45	47	51	57	59	65	69	71	75	.
13					43	47	49	53	59	61	67	71	73	77	.
17						51	53	57	63	65	71	75	77	81	.
19							55	59	65	67	73	77	79	83	.
23								63	69	71	77	81	83	87	.
29									75	77	83	87	89	93	.
31										79	85	89	91	95	.
37											91	95	97	101	.
41												99	101	105	.
43													103	107	.
47														111	.

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19 +	3	5	7	11	13	17	19	23	29	31	37	41	43	47	
3	25	27	29	33	35	39	41	45	51	53	59	63	65	69	.
5		29	31	35	37	41	43	47	53	55	61	65	67	71	.
7			33	37	39	43	45	49	55	57	63	67	69	73	.
11				41	43	47	49	53	59	61	67	71	73	77	.
13					45	49	51	55	61	63	69	73	75	79	.
17						53	55	59	65	67	73	77	79	83	.
19							57	61	67	69	75	79	81	85	.
23								65	71	73	79	83	85	89	.
29									77	79	85	89	91	95	.
31										81	87	91	93	97	.
37											93	97	99	103	.
41												101	103	107	.
43													105	109	.
47														113	.

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23 +	3	5	7	11	13	17	19	23	29	31	37	41	43	47	
3	29	31	33	37	39	43	45	49	55	57	63	67	69	73	.
5		33	35	39	41	45	47	51	57	59	65	69	71	75	.
7			37	41	43	47	49	53	59	61	67	71	73	77	.
11				45	47	51	53	57	63	65	71	75	77	81	.
13					49	53	55	59	65	67	73	77	79	83	.
17						57	59	63	69	71	77	81	83	87	.
19							61	65	71	73	79	83	85	89	.
23								69	75	77	83	87	89	93	.
29									81	83	89	93	95	99	.
31										85	91	95	97	101	.
37											97	101	103	107	.
41												105	107	111	.
43													109	113	.
47														117	.

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29 +	3	5	7	11	13	17	19	23	29	31	37	41	43	47	. . .
3	35	37	39	43	45	49	51	55	61	63	69	73	75	79	.
5		39	41	45	47	51	53	57	63	65	71	75	77	81	.
7			43	47	49	53	55	59	65	67	73	77	79	83	.
11				51	53	57	59	63	69	71	77	81	83	87	.
13					55	59	61	65	71	73	79	83	85	89	.
17						63	65	69	75	77	83	87	89	93	.
19							67	71	77	79	85	89	91	95	.
23								75	81	83	89	93	95	99	.
29									87	89	95	99	101	105	.
31										91	97	101	103	107	.
37											103	107	109	113	.
41												111	113	117	.
43													115	119	.
47														123	.

. . .

31 +	3	5	7	11	13	17	19	23	29	31	37	41	43	47	. . .
3	37	39	41	45	47	51	53	57	63	65	71	75	77	81	.
5		41	43	47	49	53	55	59	65	67	73	77	79	83	.
7			45	49	51	55	57	61	67	69	75	79	81	85	.
11				53	55	59	61	65	71	73	79	83	85	89	.
13					57	61	63	67	73	75	81	85	87	91	.
17						65	67	71	77	79	85	89	91	95	.
19							69	73	79	81	87	91	93	97	.
23								77	83	85	91	95	97	101	.
29									89	91	97	101	103	107	.
31										93	99	103	105	109	.
37											105	109	111	115	.
41												113	115	119	.
43													117	121	.
47														125	.

. . .

37 +	3	5	7	11	13	17	19	23	29	31	37	41	43	47
3	43	45	47	51	53	57	59	63	69	71	77	81	83	87
5		47	49	53	55	59	61	65	71	73	79	83	85	89
7			51	55	57	61	63	67	73	75	81	85	87	91
11				59	61	65	67	71	77	79	85	89	91	95
13					63	67	69	73	79	81	87	91	93	97
17						71	73	77	83	85	91	95	97	101
19							75	79	85	87	93	97	99	103
23								83	89	91	97	101	103	107
29									95	97	103	107	109	113
31										99	105	109	111	115
37											111	115	117	121
41												119	121	125
43													123	127
47														131

41 +	3	5	7	11	13	17	19	23	29	31	37	41	43	47
3	47	49	51	55	57	61	63	67	73	75	81	85	87	91
5		51	53	57	59	63	65	69	75	77	83	87	89	93
7			55	59	61	65	67	71	77	79	85	89	91	95
11				63	65	69	71	75	81	83	89	93	95	99
13					67	71	73	77	83	85	91	95	97	101
17						75	77	81	87	89	95	99	101	105
19							79	83	89	91	97	101	103	107
23								87	93	95	101	105	107	111
29									99	101	107	111	113	117
31										103	109	113	115	119
37											115	119	121	125
41												123	125	129
43													127	131
47														135

43 +	3	5	7	11	13	17	19	23	29	31	37	41	43	47
3	49	51	53	57	59	63	65	69	75	77	83	87	89	93
5		53	55	59	61	65	67	71	77	79	85	89	91	95
7			57	61	63	67	69	73	79	81	87	91	93	97
11				65	67	71	73	77	83	85	91	95	97	101
13					69	73	75	79	85	87	93	97	99	103
17						77	79	83	89	91	97	101	103	107
19							81	85	91	93	99	103	105	109
23								89	95	97	103	107	109	113
29									101	103	109	113	115	119
31										105	111	115	117	121
37											117	121	123	127
41												125	127	131
43													129	133
47														137

47 +	3	5	7	11	13	17	19	23	29	31	37	41	43	47
3	53	55	57	61	63	67	69	73	79	81	87	91	93	97
5		57	59	63	65	69	71	75	81	83	89	93	95	99
7			61	65	67	71	73	77	83	85	91	95	97	101
11				69	71	75	77	81	87	89	95	99	101	105
13					73	77	79	83	89	91	97	101	103	107
17						81	83	87	93	95	101	105	107	111
19							85	89	95	97	103	107	109	113
23								93	99	101	107	111	113	117
29									105	107	113	117	119	123
31										109	115	119	121	125
37											121	125	127	131
41												129	131	135
43													133	137
47														141

UNSOLVED PROBLEM: 105

Second table sequence:

0,0,0,0,1,2,4,4,6,7,9,10,11,15,17,16,19,19,23,25,26,26,28,

33,32,35,43,39,40,43,43,...

(a(2k+1) represents the number of different combinations such that 2k+1 is written as a sum of three odd primes.)

This sequence is deduced from the second table.

Study the second table and the second table sequence.

References:

Florentin Smarandache, "Problems with and without ... problems!", Ed. Somipress, Fes, Morocco, 1983;

Arizona State University, Hayden Library, "The Florentin Smarandache papers" special collection, Tempe, AZ 85287-1006, USA.

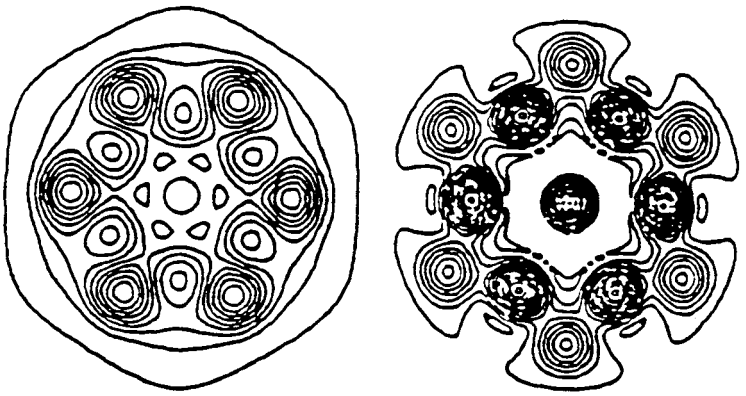
Contents

Mathematical Philosophy? ...instead of preface 3
Unsolved Problem 1 8
Unsolved Problem 2 8
Unsolved Problem 3 9
Unsolved Problem 4 10
Unsolved Problem 5 11
Unsolved Problem 6 12
Unsolved Problem 7 13
Unsolved Problem 8 14
Unsolved Problem 9 15
Unsolved Problem 10 15
Unsolved Problem 11 16
Unsolved Problem 12 16
Unsolved Problem 13 17
Unsolved Problem 14 17
Unsolved Problem 15 18
Unsolved Problem 16 18
Unsolved Problem 17 19
Unsolved Problem 18 20
Unsolved Problem 19 20
Unsolved Problem 20 21
Unsolved Problem 21 22
Unsolved Problem 22 22
Unsolved Problem 23 23
Unsolved Problem 24 24
Unsolved Problem 25 24
Unsolved Problem 26 25
Unsolved Problem 27 25
Unsolved Problem 28 26
Unsolved Problem 29 26
Unsolved Problem 30 27
Unsolved Problem 31 27
Unsolved Problem 32 28
Unsolved Problem 33 30
Unsolved Problem 34 30
Unsolved Problem 35 31
Unsolved Problem 36 31
Unsolved Problem 37 32
Unsolved Problem 38 32
Unsolved Problem 39 33
Unsolved Problem 40 34
Unsolved Problem 41 35
Unsolved Problem 42 36
Unsolved Problem 43 37
Unsolved Problem 44 38
Unsolved Problem 45 39
Unsolved Problem 46 40

Unsolved Problem 47 41
Unsolved Problem 48 42
Unsolved Problem 49 42
Unsolved Problem 50 43
Unsolved Problem 51 43
Unsolved Problem 52 44
Unsolved Problem 53 44
Unsolved Problem 54 45
Unsolved Problem 55 46
Unsolved Problem 56 47
Unsolved Problem 57 48
Unsolved Problem 58 48
Unsolved Problem 59 49
Unsolved Problem 60 49
Unsolved Problem 61 50
Unsolved Problem 62 51
Unsolved Problem 63 52
Unsolved Problem 64 53
Unsolved Problem 65 54
Unsolved Problem 66 54
Unsolved Problem 67 55
Unsolved Problem 68 55
Unsolved Problem 69 56
Unsolved Problem 70 57
Unsolved Problem 71 58
Unsolved Problem 72 58
Unsolved Problem 73 59
Unsolved Problem 74 60
Unsolved Problem 75 62
Unsolved Problem 76 63
Unsolved Problem 77 64
Unsolved Problem 78 65
Unsolved Problem 79 65
Unsolved Problem 80 67
Unsolved Problem 81 68
Unsolved Problem 82 68
Unsolved Problem 83 69
Unsolved Problem 84 70
Unsolved Problem 85 70
Unsolved Problem 86 72
Unsolved Problem 87 73
Unsolved Problem 88 75
Unsolved Problem 89 78
Unsolved Problem 90 82
Unsolved Problem 91 83
Unsolved Problem 92 84
Unsolved Problem 93 85
Unsolved Problem 94 86
Unsolved Problem 95 87
Unsolved Problem 96 88
Unsolved Problem 97 89
Unsolved Problem 98 90
Unsolved Problem 99 91
Unsolved Problem 100 92

Unsolved Problem 101 94
Unsolved Problem 102 95
Unsolved Problem 103 98
Unsolved Problem 104 100
Unsolved Problem 105 108

easy easy problems



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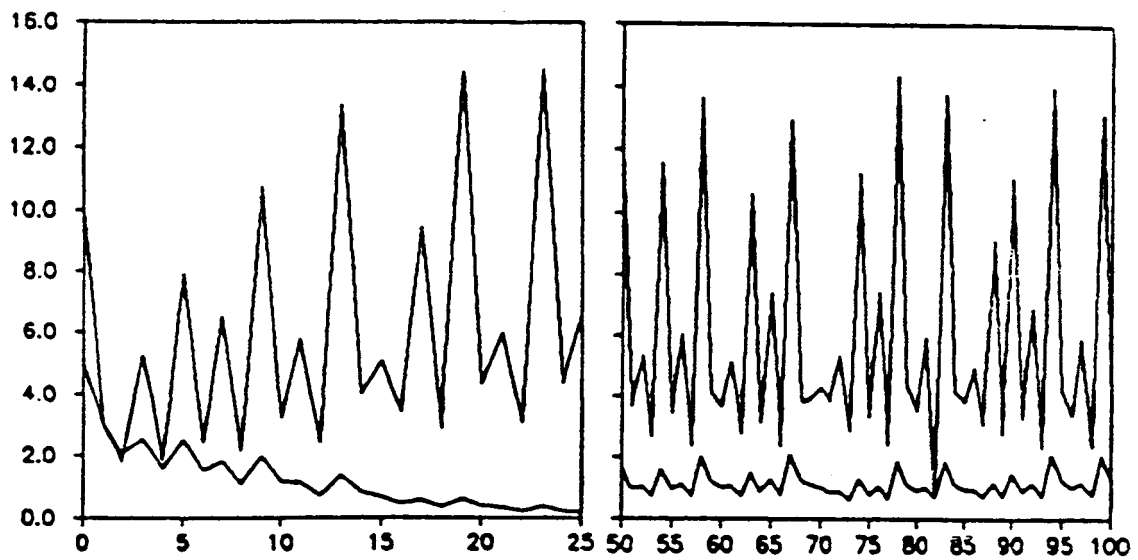

still still unsolved!

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